SISTEM ANTRIAN $M/M/c/\text{GD}/\infty/\infty$

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Sistem antrian dengan waktu antardatang berdistribusi eksponensial atau jumlah pelanggan yang datang berdistribusi Poisson. Waktu layannya berdistribusi eksponensial atau jumlah pelanggan yang berangkat berdistribusi Poisson. Jumlah pelayan paralelnya sebanyak *c*. Disiplin pelayanannya umum. Jumlah pelanggan maksimum yang diperbolehkan dalam sistem sebanyak takhingga. Jumlah populasi pelanggan takhingga. Sistem antrian ini dinotasikan dengan sistem antrian $M/M/c/\text{GD}/\infty/\infty$.

Laju datang

$$\lambda_n = \lambda \qquad \text{konstan untuk} \qquad n \geq 0$$

di mana *lamda* menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Lamda n menyatakan laju datang keadaan jumlah pelanggan sebanyak *n*.

Laju layan

$$\mu_n = \begin{cases} n\,\mu & \text{if } n \leq c \\ c\,\mu & \text{if } n \geq c \end{cases}$$

Mu menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

Mu n menyatakan laju layan keadaan jumlah pelanggan sebanyak *n*.

Laju datang rata-rata effektif

$$\lambda_{eff} = \sum_{n=0}^{\infty} \left(\lambda_n\, p_n\right) = \sum_{n=0}^{\infty} \left(\lambda\ p_n\right) = \lambda \sum_{n=0}^{\infty} p_n = \lambda\ (1) = \lambda$$

Di mana p_n menyatakan probabilitas ada *n* pelanggan sistem antrian dalam keadaan mapan (*steady state*), menyatakan juga ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan sebanyak *n*.

Keadaan mapan (*steady state*) berarti distribusi probabilitas jumlah pelanggan dalam antrian dan distribusi probabilitas jumlah pelanggan dalam sistem tidak bergantung waktu.

Jika $n \leq c$

$$p_n = \frac{\lambda_{n-1}\, \lambda_{n-2} \cdots \lambda_0}{\mu_n\, \mu_{n-1} \cdots \mu_1}\, p_0 = \frac{\lambda\ \lambda\ \ldots\ \lambda}{(n\,\mu)\ [(n-1)\ \mu]\ \ldots\ \mu}\, p_0 = \frac{\lambda^n}{n!\ \mu^n}\, p_0$$

$$p_n = \frac{\lambda^n}{n!\ \mu^n}\, p_0 = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0$$

Jika $n \geq c$

$$p_n = \frac{\lambda_{n-1}\ \lambda_{n-2}\ \cdots\ \lambda_0}{\mu_n\ \mu_{n-1}\ \cdots\ \mu_1}\, p_0 = \frac{\lambda^n}{\mu_n\ \mu_{n-1}\ \cdots\ \mu_{c+1}\ \mu_c\ \mu_{c-1}\ \cdots\ \mu_2\ \mu_1}\, p_0$$

$$p_n = \frac{\lambda^n}{(c\,\mu)\ (c\,\mu)\ \ldots\ (c\,\mu)\ (c\,\mu)\ [(c-1)\ \mu]\ \ldots\ \mu_2\ \mu_1}\, p_0$$

$$p_n = \frac{\lambda^n}{(c\,\mu)\ (c\,\mu)\ \ldots\ (c\,\mu)\ (c\,\mu)\ [(c-1)\ \mu]\ \ldots\ (2\,\mu)\ (\mu)}\, p_0$$

$$p_n = \frac{\lambda^n}{(c\,\mu)^{n-c}\ (c\,\mu)\ [(c-1)\ \mu]\ \ldots\ (2\,\mu)\ (\mu)}\, p_0$$

$$p_n = \frac{\lambda^n}{(c\,\mu)^{n-c}\left[[c\ (c-1)\ \ldots\ (2)\ 1]\ \mu^c\right]}\, p_0$$

$$p_n = \frac{\lambda^n}{(c\,\mu)^{n-c}\left(c!\ \mu^c\right)}\, p_0 = \frac{\lambda^n}{\left(c^{n-c}\ \mu^{n-c}\right)\left(c!\ \mu^c\right)}\, p_0 = \frac{\lambda^n}{c!\ c^{n-c}\ \mu^n}\, p_0$$

$$p_n = \frac{1}{c!\ c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n p_0$$

Untuk mencari p_0 didapat dari

$$\sum_{n=0}^{\infty} p_n = 1$$

$$\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0\right] + \sum_{n=c}^{\infty}\left[\frac{1}{c!\ c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n p_0\right] = 1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \sum_{n=c}^{\infty}\left[\frac{1}{c!\ c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n\right]\right] = 1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\sum_{n-c=c-c}^{\infty-c}\left[\frac{1}{c!\,c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n\right]\right]=1$$

Bila ditulis $n-c=m$ atau $n=m+c$ maka

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\sum_{m=0}^{\infty}\left[\frac{1}{c!\,c^{m}}\left(\frac{\lambda}{\mu}\right)^{m+c}\right]\right]=1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\sum_{m=0}^{\infty}\left[\frac{1}{c^{m}}\left(\frac{\lambda^m\,\lambda^c}{\mu^m\,\mu^c}\right)\right]\right]=1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\sum_{m=0}^{\infty}\left[\frac{\lambda^m}{c^{m}\,\mu^m}\left(\frac{\lambda^c}{\mu^c}\right)\right]\right]=1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\sum_{m=0}^{\infty}\left[\left(\frac{\lambda}{c\,\mu}\right)^m\left(\frac{\lambda}{\mu}\right)^c\right]\right]=1$$

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\sum_{m=0}^{\infty}\left[\left(\frac{\lambda}{c\,\mu}\right)^m\right]\right]=1$$

padahal $$\sum_{m=0}^{\infty}\left[\left(\frac{\lambda}{c\,\mu}\right)^m\right]=\frac{1}{1-\dfrac{\lambda}{c\,\mu}}\qquad \text{untuk } \frac{\lambda}{c\,\mu}<1$$

maka

$$p_0\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{1}{1-\dfrac{\lambda}{c\,\mu}}\right)\right]=1\qquad \text{untuk } \frac{\lambda}{c\,\mu}<1$$

jadi

$$p_0 = \frac{1}{\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{1}{1-\frac{\lambda}{c\,\mu}}\right)\right]} \qquad \text{untuk } \frac{\lambda}{c\,\mu} < 1$$

atau

$$p_0 = \frac{1}{\left[\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]} \qquad \text{untuk } \frac{\lambda}{c\,\mu} < 1$$

$$\rho = \frac{\lambda}{c\,\mu}$$ **dikenal sebagai faktor utilisasi / intensitas lalu lintas.**

Probabilitas ada n pelanggan dalam sistem antrian yang keadaannya mapan

$$p_n = \begin{cases} \text{if } (n \le c) \wedge \left(0 < \frac{\lambda}{c\,\mu} < 1\right) \\ \quad \left| \begin{array}{l} p_0 \leftarrow \dfrac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n p_0 \end{array}\right. \\ \text{if } (n \ge c) \wedge \left(0 < \frac{\lambda}{c\,\mu} < 1\right) \\ \quad \left| \begin{array}{l} p_0 \leftarrow \dfrac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ \dfrac{1}{c!\,c^{n-c}}\left(\dfrac{\lambda}{\mu}\right)^n p_0 \end{array}\right. \\ \text{"Tidak didefinisikan"} \quad \textit{otherwise} \end{cases}$$

Probabilitas ini fungsi dari (n, λ, μ, c) sehingga dapat dinotasikan secara lengkap sebagai:

$$p(n,\lambda,\mu,c) := \left|\begin{array}{l} n \leftarrow \dfrac{n}{pelanggan} \\ c \leftarrow \dfrac{c}{pelayan} \\ \text{if } (n \le c) \wedge \left(0 < \dfrac{\lambda}{c\,\mu} < 1\right) \\ \quad \left|\begin{array}{l} p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n p_0 \end{array}\right. \\ \text{if } (n \ge c) \wedge \left(0 < \dfrac{\lambda}{c\,\mu} < 1\right) \\ \quad \left|\begin{array}{l} p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ \dfrac{1}{c!\,c^{n-c}}\left(\dfrac{\lambda}{\mu}\right)^n p_0 \end{array}\right. \\ \text{"Tidak didefinisikan"} \quad \textit{otherwise} \end{array}\right.$$

Contoh 3.1

$\lambda \equiv 14 \dfrac{pelanggan}{jam}$

laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \sum_{n=0}^{\infty} (\lambda_n p_n)$.

$\lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \ge 0$.

$\mu \equiv 4 \dfrac{pelanggan}{jam}$

laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

$c := 5 pelayan$

$\lambda = 14 \dfrac{pelanggan}{jam}$ $\qquad$ $\mu = 4 \dfrac{pelanggan}{jam}$

Bila $n := 0 pelanggan$ $\;p(n, \lambda, \mu, c) = 0.026$ angka ini juga menyatakan ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan $n = 0$.

Bila $n := 7 pelanggan$ $\;p(n, \lambda, \mu, c) = 0.056$ angka ini juga menyatakan ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan $n = 7 pelanggan$.

$p(0 pelanggan, \lambda, \mu, c) = 0.026$
$p(1 pelanggan, \lambda, \mu, c) = 0.091$
$p(2 pelanggan, \lambda, \mu, c) = 0.159$
$p(3 pelanggan, \lambda, \mu, c) = 0.185$
$p(4 pelanggan, \lambda, \mu, c) = 0.162$
$p(5 pelanggan, \lambda, \mu, c) = 0.113$
$p(6 pelanggan, \lambda, \mu, c) = 0.079$
$p(7 pelanggan, \lambda, \mu, c) = 0.056$
$p(8 pelanggan, \lambda, \mu, c) = 0.039$
$p(9 pelanggan, \lambda, \mu, c) = 0.027$
$p(10 pelanggan, \lambda, \mu, c) = 0.019$
$p(11 pelanggan, \lambda, \mu, c) = 0.013$

Jumlah pelayan minimum:

$$c_{min}(\lambda, \mu) \equiv \left| \begin{array}{l} \left(ceil\left(\dfrac{\lambda}{\mu}\right) + 1\right) pelayan \quad if \;\; ceil\left(\dfrac{\lambda}{\mu}\right) = \dfrac{\lambda}{\mu} \\ \left(ceil\left(\dfrac{\lambda}{\mu}\right)\right) pelayan \quad otherwise \end{array} \right.$$

$c_{min}(\lambda, \mu) = 4 pelayan$

$ORIGIN \equiv \dfrac{c_{min}(\lambda, \mu)}{pelayan}$

$c_{atas}(\lambda, \mu) \equiv 3\, c_{min}(\lambda, \mu)$ $\qquad$ sebagai contoh saja

$c := c_{min}(\lambda, \mu), \left(c_{min}(\lambda, \mu) + 1\, pelayan\right) .. c_{atas}(\lambda, \mu)$ $\qquad$ jumlah pelayan.

$c_{min}(\lambda, \mu) = 4 pelayan$ $\qquad$ $c_{atas}(\lambda, \mu) = 12 pelayan$

Faktor utilisasi / intensitas lalu lintas:

$$\rho(\lambda,\mu,c) := \frac{\lambda}{c\,\mu}$$

$c =$ (pelayan)	$\rho(\lambda,\mu,c) =$ $\left(\frac{1}{pelayan}\right)$
4	0.875
5	0.7
6	0.583
7	0.5
8	0.438
9	0.389
10	0.35
11	0.318
12	0.292

Ekspektasi jumlah pelanggan dalam antrian atau ekspektasi jumlah pelanggan antri

Terjadi antrian jika $n \geq c$ dengan

$$p_n = \frac{1}{c!\; c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n p_0$$

maka ekspektasi jumlah pelanggan dalam antrian atau ekspektasi jumlah pelanggan antri:

$$EN_q = \sum_{n=c}^{\infty} \left[(n-c)p_n\right]$$

$$EN_q = \sum_{n=c}^{\infty} \left[(n-c)\left[\frac{1}{c!\; c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n p_0\right]\right]$$

$$EN_q = \frac{1}{c!}\, p_0 \sum_{n=c}^{\infty} \left[(n-c)\left[\frac{1}{c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n\right]\right]$$

$$EN_q = \frac{1}{c!}\, p_0 \sum_{n-c=c-c}^{\infty-c} \left[(n-c)\left[\frac{1}{c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n\right]\right]$$

$$EN_q = \frac{1}{c!}\, p_0 \sum_{n-c=0}^{\infty}\left[(n-c)\left[\frac{1}{c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n\right]\right]$$

Bila ditulis $n-c = m$ atau $n = m+c$ maka

$$EN_q = \frac{1}{c!}\, p_0 \sum_{m=0}^{\infty}\left[m\left[\frac{1}{c^{m}}\left(\frac{\lambda}{\mu}\right)^{m+c}\right]\right]$$

$$EN_q = \frac{1}{c!}\, p_0 \sum_{m=0}^{\infty}\left[m\left[\frac{1}{c^{m}}\left(\frac{\lambda}{\mu}\right)^{m}\left(\frac{\lambda}{\mu}\right)^{c}\right]\right]$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c} p_0 \sum_{m=0}^{\infty}\left[m\left[\frac{1}{c^{m}}\left(\frac{\lambda}{\mu}\right)^{m}\right]\right]$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c} p_0 \sum_{m=0}^{\infty}\left[m\left[\left(\frac{\lambda}{c\,\mu}\right)^{m}\right]\right]$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c} p_0 \sum_{m=0}^{\infty}\left[m\left[\left(\frac{\lambda}{c\,\mu}\right)^{m-1}\left(\frac{\lambda}{c\,\mu}\right)\right]\right]$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c}\left(\frac{\lambda}{c\,\mu}\right) p_0 \sum_{m=0}^{\infty}\left[m\left(\frac{\lambda}{c\,\mu}\right)^{m-1}\right]$$

padahal $$m\left(\frac{\lambda}{c\,\mu}\right)^{m-1} = \frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\left(\frac{\lambda}{c\,\mu}\right)^{m}$$

maka $$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c}\left(\frac{\lambda}{c\,\mu}\right) p_0 \sum_{m=0}^{\infty}\left[\frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\left(\frac{\lambda}{c\,\mu}\right)^{m}\right]$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^{c}\left(\frac{\lambda}{c\,\mu}\right) p_0 \frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\sum_{m=0}^{\infty}\left(\frac{\lambda}{c\,\mu}\right)^{m}$$

padahal $$\sum_{m=0}^{\infty}\left(\frac{\lambda}{c\,\mu}\right)^{m} = \frac{1}{1-\frac{\lambda}{c\,\mu}}$$ untuk $\frac{\lambda}{c\,\mu} < 1$

maka $$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{\lambda}{c\,\mu}\right)p_0\,\frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\left(\frac{1}{1-\frac{\lambda}{c\,\mu}}\right)$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{\lambda}{c\,\mu}\right)p_0\,\frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\left(\frac{1}{1-\frac{\lambda}{c\,\mu}}\right)$$

padahal $$\frac{d}{d\left(\frac{\lambda}{c\,\mu}\right)}\left(\frac{1}{1-\frac{\lambda}{c\,\mu}}\right) = \frac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2}$$

maka

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{\lambda}{c\,\mu}\right)p_0\,\frac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2} = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{\lambda}{c\,\mu}\right)\frac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2}\,p_0$$

$$EN_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{\lambda}{c\,\mu}\right)\frac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2}\;\frac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)}$$

Ekspektasi jumlah pelanggan antri ini fungsi dari (λ, μ, c) sehingga dapat dinotasikan secara lengkap sebagai:

$$EN_q(\lambda,\mu,c) := \left|\begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\ \quad\left|\begin{array}{l} p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ ENq \leftarrow \dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c\dfrac{\lambda}{c\,\mu}\,\dfrac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2}\,p_0 \\ ENq\ pelanggan \end{array}\right. \\ \text{"Tidak didefinisikan"} \quad otherwise \end{array}\right.$$

Contoh 3.2

$\lambda = 14 \dfrac{pelanggan}{jam}$ laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \sum_{n=0}^{\infty} (\lambda_n p_n)$. $\lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$.

$\mu = 4 \dfrac{pelanggan}{jam}$ laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

Jumlah pelayan — Ekspektasi jumlah pelanggan antri

$c =$ (*pelayan*)	$EN_q(\lambda, \mu, c) =$ (*pelanggan*)
4	5.165
5	0.882
6	0.248
7	0.076
8	0.023
9	$6.824 \cdot 10^{-3}$
10	$1.901 \cdot 10^{-3}$
11	$4.999 \cdot 10^{-4}$
12	$1.238 \cdot 10^{-4}$

Ekspektasi waktu antri

Ekspektasi waktu antri yaitu waktu rata-rata pelanggan berada dalam antrian

$$ED(\lambda, \mu, c) = \frac{EN_q(\lambda, \mu, c)}{\lambda_{eff}}$$

$$
ED(\lambda, \mu, c) := \left|
\begin{array}{l}
c \leftarrow \dfrac{c}{pelayan} \\
\text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\
\quad \left|
\begin{array}{l}
\lambda_{eff} \leftarrow \lambda \\
p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu - \lambda}\right)} \\
ENq \leftarrow \dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\,\mu}\,\dfrac{1}{\left(1 - \dfrac{\lambda}{c\,\mu}\right)^2}\,p_0 \\
\dfrac{ENq\ pelanggan}{\lambda_{eff}}
\end{array}
\right. \\
\text{"Tidak didefinisikan"} \quad \text{otherwise}
\end{array}
\right.
$$

Contoh 3.3

$\lambda = 14\,\dfrac{pelanggan}{jam}$

laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \displaystyle\sum_{n=0}^{\infty}\left(\lambda_n\, p_n\right)$.

$\lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$.

$\mu = 4\,\dfrac{pelanggan}{jam}$

laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

Jumlah pelayan	Ekspektasi waktu antri yaitu waktu rata-rata pelanggan berada dalam antrian
$c =$	$ED(\lambda, \mu, c) =$
4 *pelayan*	0.369 *jam*
5	0.063
6	0.018
7	$5.443 \cdot 10^{-3}$
8	$1.66 \cdot 10^{-3}$
9	$4.874 \cdot 10^{-4}$
10	$1.358 \cdot 10^{-4}$
11	$3.571 \cdot 10^{-5}$
12	$8.846 \cdot 10^{-6}$

Ekspektasi waktu sistem

Ekspektasi waktu sistem yaitu waktu rata-rata pelanggan berada dalam sistem.

$$EW(\lambda, \mu, c) = ED(\lambda, \mu, c) + \frac{1}{\mu}$$

$$ED(\lambda, \mu, c) = \frac{EN_q(\lambda, \mu, c)}{\lambda_{eff}}$$

$$EW(\lambda, \mu, c) := \left| \begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\ \quad \left| \begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1} \left[\frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu - \lambda}\right)} \\ ENq \leftarrow \dfrac{1}{c!} \left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\,\mu} \dfrac{1}{\left(1 - \dfrac{\lambda}{c\,\mu}\right)^2} p_0 \\ \dfrac{ENq\ pelanggan}{\lambda_{eff}} + \dfrac{1 pelanggan}{\mu} \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad otherwise \end{array} \right.$$

Contoh 3.4

$\lambda = 14\,\dfrac{pelanggan}{jam}$

laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \sum_{n=0}^{\infty} (\lambda_n p_n)$.

$\lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$.

$\mu = 4\,\dfrac{pelanggan}{jam}$

laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

Jumlah pelayan

Ekspektasi waktu sistem yaitu waktu rata-rata pelanggan berada dalam sistem

$c =$ (*pelayan*)	$EW(\lambda, \mu, c) =$ (*jam*)
4	0.619
5	0.313
6	0.268
7	0.255
8	0.252
9	0.25
10	0.25
11	0.25
12	0.25

Ekspektasi jumlah pelanggan sistem

Ekspektasi jumlah pelanggan sistem besarnya

$$EN(\lambda,\mu,c) = \lambda_{eff}\ EW(\lambda,\mu,c)$$

$$EN(\lambda,\mu,c) := \left| \begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\ \mu} < 1 \\ \quad \left| \begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1} \left[\dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n\right] + \dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \left(\dfrac{c\ \mu}{c\ \mu - \lambda}\right)} \\ ENq \leftarrow \dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\ \mu}\ \dfrac{1}{\left(1-\dfrac{\lambda}{c\ \mu}\right)^2}\ p_0 \\ \lambda_{eff}\left(\dfrac{ENq\ pelanggan}{\lambda_{eff}} + \dfrac{1 pelanggan}{\mu}\right) \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad otherwise \end{array} \right.$$

Contoh 3.5

$\lambda = 14\ \dfrac{pelanggan}{jam}$ laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \displaystyle\sum_{n=0}^{\infty} (\lambda_n\ p_n)$. $\quad \lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$.

$\mu = 4\ \dfrac{pelanggan}{jam}$ laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang dilayani rata-rata per satuan waktu.

Jumlah pelayan $c =$		Ekspektasi jumlah pelanggan sistem $EN(\lambda, \mu, c) =$	
4	*pelayan*	8.665	*pelanggan*
5		4.382	
6		3.748	
7		3.576	
8		3.523	
9		3.507	
10		3.502	
11		3.5	
12		3.5	