MODEL KEPUTUSAN ANTRIAN $M/M/c/GD/\infty/\infty$
MODEL ONGKOS

JUMLAH PELAYAN OPTIMAL

$\lambda \equiv 125 \frac{pelanggan}{jam}$

laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif . $\lambda_{eff} = \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$.

$\mu \equiv 10 \frac{pelanggan}{jam}$

laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang telah dilayani rata-rata per satuan waktu.

$O_1 \equiv 11000 \frac{Rp}{jam \ \ pelayan}$

ongkos pelayanan per pelayan per satuan waktu.

$O_2 \equiv 25000 \frac{Rp}{jam \ \ pelanggan}$

ongkos (*nilai*) per pelanggan dalam sistem per satuan waktu.

Jumlah pelayan minimum:

$$c_{min}(\lambda, \mu) \equiv \left| \begin{array}{ll} \left(ceil\left(\frac{\lambda}{\mu}\right) + 1\right) pelayan & if \ ceil\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ \left(ceil\left(\frac{\lambda}{\mu}\right)\right) pelayan & otherwise \end{array} \right.$$

$c_{min}(\lambda, \mu) = 13 \, pelayan$

$ORIGIN \equiv \frac{c_{min}(\lambda, \mu)}{pelayan}$

$c_{atas}(\lambda, \mu) \equiv 3 \, c_{min}(\lambda, \mu)$

$c = c_{min}(\lambda, \mu), \left(c_{min}(\lambda, \mu) + 1 \ pelayan\right) .. c_{atas}(\lambda, \mu)$ jumlah pelayan.

$c_{min}(\lambda, \mu) = 13 \, pelayan$

$c_{atas}(\lambda, \mu) = 39 \, pelayan$

Faktor utilisasi / intensitas lalu lintas:

$$\rho(\lambda, \mu, c) = \frac{\lambda}{c \, \mu}$$

Probabilitas ada nol pelanggan dalam sistem antrian yang keadaannya mapan (*steady state*), juga menyatakan juga ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan nol atau sistem sedang menganggur.
Keadaannya mapan (*steady state*) berarti distribusi probabilitas jumlah pelanggan dalam antrian dan distribusi probabilitas jumlah pelanggan dalam sistem tidak bergantung waktu.

Probabilitas ada n pelanggan dalam sistem antrian yang keadaannya mapan (*steady state*), juga menyatakan juga ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan n.

Ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan c:

$$EOT_{MMcGD}(\lambda, \mu, c, O_1, O_2) = \begin{cases} EOO_{MMcGD}(c, O_1) + EON_{MMcGD}(\lambda, \mu, c, O_2) & \text{if } 0 < \frac{\lambda}{c\ \mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases}$$

Ekspektasi ongkos operasi para pelayan per satuan waktu untuk jumlah pelayan c:

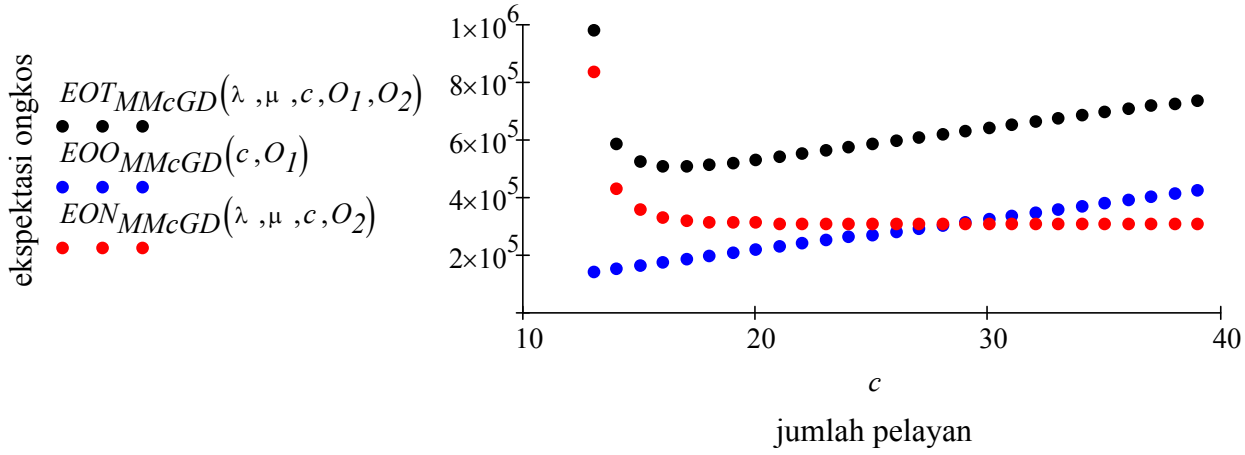
$$EOO_{MMcGD}(c, O_1) = \begin{cases} c\ O_1 & \text{if } 0 < \frac{\lambda}{\frac{c}{pelayan}\ \mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases}$$

Ekspektasi ongkos para pelanggan berada dalam sistem per satuan waktu untuk jumlah pelayan c:

$$EON_{MMcGD}(\lambda, \mu, c, O_2) = \begin{cases} c \leftarrow \frac{c}{pelayan} \\ \text{if } 0 < \frac{\lambda}{c\ \mu} < 1 \\ \quad \begin{cases} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \frac{1}{\sum_{n=0}^{c-1} \left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\ \mu}{c\ \mu - \lambda}\right)} \\ EkspN \leftarrow \lambda_{eff}\left[\frac{1}{\lambda_{eff}}\left[\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda}{c\ \mu} \frac{1}{\left(1 - \frac{\lambda}{c\ \mu}\right)^2} p_0\right] + \frac{1}{\mu}\right] \\ O_2\ EkspN\ pelanggan \end{cases} \\ \text{"Tidak didefinisikan"} \quad \text{otherwise} \end{cases}$$

Jadi ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan c:

$$EOT_{MMcGD}(\lambda, \mu, c, O_1, O_2) = \left|\begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\ \mu} < 1 \\ \quad \left|\begin{array}{l} EOO \leftarrow c\ pelayan\ O_1 \\ EkspON \leftarrow \left|\begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\ \mu}{c\ \mu - \lambda}\right)} \\ EN \leftarrow \left|\begin{array}{l} ENq \leftarrow \dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\ \mu} \dfrac{1}{\left(1 - \frac{\lambda}{c\ \mu}\right)^2} p_0 \\ ED \leftarrow \dfrac{1}{\lambda_{eff}}\ ENq \\ EW \leftarrow ED + \dfrac{1}{\mu} \\ EN \leftarrow \lambda_{eff}\ EW \end{array}\right. \\ O_2\ EN \end{array}\right. \\ EOO + EkspON\ pelanggan \\ \text{"Tidak didefinisikan"}\ \ otherwise \end{array}\right.$$



Kurva ekspektasi ongkos vs. jumlah pelayan

Ekspektasi jumlah pelanggan dalam sistem:

$$EN_{MMcGD}(\lambda,\mu,c) = \left|\begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\ \quad \left|\begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)} \\ EkspN \leftarrow \lambda_{eff}\left[\dfrac{1}{\lambda_{eff}}\left[\dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\,\mu}\dfrac{1}{\left(1-\frac{\lambda}{c\,\mu}\right)^2}p_0\right]+\dfrac{1}{\mu}\right] \\ EkspN\ pelanggan \end{array}\right. \\ \text{"Tidak didefinisikan"}\ otherwise \end{array}\right.$$

$$O_1 = 1.1\times10^4\ \frac{Rp}{jam\ \ pelayan} \qquad O_2 = 2.5\times10^4\ \frac{Rp}{jam\ \ pelanggan}$$

$c =$	$EOO_{MMcGD}(c,O_1)$	$EN_{MMcGD}(\lambda,\mu,c)$	$EOT_{MMcGD}(\lambda,\mu,c,O_1,O_2)$
13 *pelayan*	$1.43\cdot10^5$ $\frac{Rp}{jam}$	33.626 *pelanggan*	$9.837\cdot10^5$ $\frac{Rp}{jam}$
14	$1.54\cdot10^5$	17.425	$5.896\cdot10^5$
15	$1.65\cdot10^5$	14.507	$5.277\cdot10^5$
16	$1.76\cdot10^5$	13.443	$5.121\cdot10^5$
17	$1.87\cdot10^5$	12.967	$5.112\cdot10^5$
18	$1.98\cdot10^5$	12.735	$5.164\cdot10^5$
19	$2.09\cdot10^5$	12.618	$5.245\cdot10^5$
20	$2.2\cdot10^5$	12.559	$5.34\cdot10^5$
21	$2.31\cdot10^5$	12.529	$5.442\cdot10^5$
22	$2.42\cdot10^5$	12.514	$5.548\cdot10^5$
23	$2.53\cdot10^5$	12.506	$5.657\cdot10^5$
24	$2.64\cdot10^5$	12.503	$5.766\cdot10^5$
25	$2.75\cdot10^5$	12.501	$5.875\cdot10^5$
...	...	...	...

Jumlah pelayan optimal:

Dalam program ini .

$$c_{optMMcGD}(\lambda,\mu,O_1,O_2) = \left| \begin{array}{l} c_{min} \leftarrow \left| \begin{array}{ll} ceil\left(\frac{\lambda}{\mu}\right)+1 & if\ ceil\left(\frac{\lambda}{\mu}\right)=\frac{\lambda}{\mu} \\ ceil\left(\frac{\lambda}{\mu}\right) & otherwise \end{array} \right. \\ c_{atas} \leftarrow 3\ c_{min} \\ for\ \ c \in c_{min}..c_{atas} \\ \quad v_{EOT_c} \leftarrow \left| \begin{array}{l} if\ \ 0<\frac{\lambda}{c\ \mu}<1 \\ \quad \left| \begin{array}{l} EOO \leftarrow c\ pelayan\ \ O_1 \\ \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \frac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\ \mu}{c\ \mu-\lambda}\right)} \\ EkspON \leftarrow \left| \begin{array}{l} ENq \leftarrow \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda}{c\ \mu} \frac{1}{\left(1-\frac{\lambda}{c\ \mu}\right)^2} p_0 \\ ED \leftarrow \frac{1}{\lambda_{eff}}\ ENq \\ EW \leftarrow ED+\frac{1}{\mu} \\ EN \leftarrow \lambda_{eff}\ EW \\ O_2\ EN \end{array} \right. \\ EOO + EkspON\ \ pelanggan \end{array} \right. \\ \text{"Tidak didefinisikan"}\ \ otherwise \end{array} \right. \\ \left(match\left(min\left(v_{EOT}\right),v_{EOT}\right)c_{min}\right)\ pelayan \end{array} \right.$$

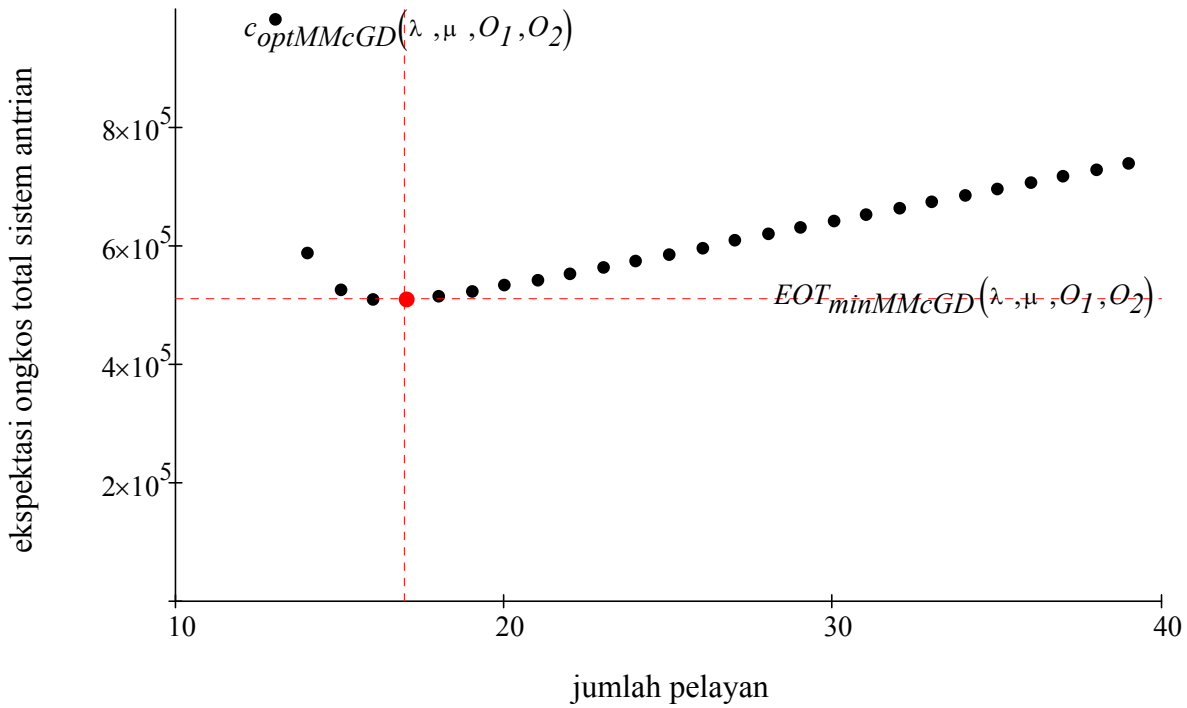
$$c_{optMMcGD}(\lambda,\mu,O_1,O_2) = 17\ pelayan$$

Ekspektasi ongkos total sistem antrian minimum:

Dalam program ini .

$$EOT_{minMMcGD}(\lambda, \mu, O_1, O_2) = \left| \begin{array}{l} c_{min} \leftarrow \left| \begin{array}{ll} ceil\left(\frac{\lambda}{\mu}\right) + 1 & if \ ceil\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ ceil\left(\frac{\lambda}{\mu}\right) & otherwise \end{array} \right. \\ c_{atas} \leftarrow 3 \ c_{min} \\ for \ \ c \in c_{min} .. c_{atas} \\ \quad v_{EOT_c} \leftarrow \left| \begin{array}{l} if \ \ 0 < \frac{\lambda}{c \ \mu} < 1 \\ \quad \left| \begin{array}{l} EOO \leftarrow c \ pelayan \ \ O_1 \\ \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \frac{1}{\sum_{n=0}^{c-1} \left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c \ \mu}{c \ \mu - \lambda}\right)} \\ EkspON \leftarrow \left| \begin{array}{l} ENq \leftarrow \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda}{c \ \mu} \frac{1}{\left(1 - \frac{\lambda}{c \ \mu}\right)^2} p_0 \\ ED \leftarrow \frac{1}{\lambda_{eff}} \ ENq \\ EW \leftarrow ED + \frac{1}{\mu} \\ EN \leftarrow \lambda_{eff} \ EW \\ O_2 \ EN \end{array} \right. \\ EOO + EkspON \ \ pelanggan \end{array} \right. \\ \text{"Tidak didefinisikan"} \ \ otherwise \end{array} \right. \\ EOT_{min} \leftarrow min(v_{EOT}) \end{array} \right.$$

$$EOT_{minMMcGD}(\lambda, \mu, O_1, O_2) = 5.112 \times 10^5 \ \frac{Rp}{jam}$$



Kurva ekspektasi ongkos total sistem antrian vs. jumlah pelayan