MODEL KEPUTUSAN ANTRIAN $M/M/c/GD/\infty/\infty$
MODEL ONGKOS

JUMLAH PELAYAN OPTIMAL

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$\lambda \equiv 125 \dfrac{pelanggan}{jam}$ laju datang pelanggan per satuan waktu.

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu.

Laju datang rata-rata effektif $\lambda_{eff} = \sum_{n=0}^{\infty} (\lambda_n p_n)$ $\lambda_{eff} := \lambda$

Dalam hal ini $\lambda_n = \lambda$ konstan untuk $n \geq 0$

$\mu \equiv 10 \dfrac{pelanggan}{jam}$ laju layan pelanggan per satuan waktu.

μ menyatakan laju layan yaitu jumlah pelanggan yang telah dilayani rata-rata per satuan waktu.

$O_1 \equiv 11000 \dfrac{Rp}{jam \;\; pelayan}$ ongkos pelayanan per pelayan per satuan waktu.

$O_2 \equiv 25000 \dfrac{Rp}{jam \;\; pelanggan}$ *ongkos* (*nilai*) per pelanggan dalam sistem per satuan waktu.

Jumlah pelayan minimum:

$$c_{min}(\lambda, \mu) \equiv \begin{cases} \left(ceil\left(\dfrac{\lambda}{\mu}\right) + 1\right) pelayan & \text{if } ceil\left(\dfrac{\lambda}{\mu}\right) = \dfrac{\lambda}{\mu} \\ \left(ceil\left(\dfrac{\lambda}{\mu}\right)\right) pelayan & \text{otherwise} \end{cases}$$

$$c_{min}(\lambda, \mu) = 13\, pelayan$$

$$ORIGIN \equiv \frac{c_{min}(\lambda, \mu)}{pelayan}$$

$$c_{atas}(\lambda, \mu) \equiv 3\, c_{min}(\lambda, \mu)$$

MODEL KEPUTUSAN ANTRIAN *M/M/c*/GD/∞/∞

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$c := c_{min}(\lambda, \mu), \left(c_{min}(\lambda, \mu) + 1\ pelayan\right) .. c_{atas}(\lambda, \mu)$ jumlah pelayan.

$$c_{min}(\lambda, \mu) = 13\,pelayan$$

$$c_{atas}(\lambda, \mu) = 39\,pelayan$$

Faktor utilisasi / intensitas lalu lintas:

$$\rho(\lambda, \mu, c) := \frac{\lambda}{c\ \mu}$$

Probabilitas ada nol pelanggan dalam sistem antrian yang keadaannya mapan (*steady state*), juga menyatakan juga ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan nol atau sistem sedang menganggur.

Keadaannya mapan (*steady state*) berarti distribusi probabilitas jumlah pelanggan dalam antrian dan distribusi probabilitas jumlah pelanggan dalam sistem tidak bergantung waktu.

Probabilitas ada *n* pelanggan dalam sistem antrian yang keadaannya mapan (*steady state*), juga menyatakan juga ekspektasi proporsi waktu bahwa sistem berada dengan jumlah pelanggan *n*.

Ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan *c*:

$$EOT_{MMcGD}(\lambda, \mu, c, O_1, O_2) = \begin{cases} EOO_{MMcGD}(c, O_1) + EON_{MMcGD}(\lambda, \mu, c, O_2) & \text{if } 0 < \dfrac{\lambda}{c\ \mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases}$$

Ekspektasi ongkos operasi para pelayan per satuan waktu untuk jumlah pelayan *c*:

$$EOO_{MMcGD}(c, O_1) := \begin{cases} c\ O_1 & \text{if } 0 < \dfrac{\lambda}{\dfrac{c}{pelayan}\ \mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases}$$

Ekspektasi ongkos para pelanggan berada dalam sistem per satuan waktu untuk jumlah pelayan *c*:

$$EON_{MMcGD}(\lambda, \mu, c, O_2) := \begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\ \quad \left| \begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1} \left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu - \lambda}\right)} \\ EkspN \leftarrow \lambda_{eff}\left[\dfrac{1}{\lambda_{eff}}\left[\dfrac{1}{c!}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\,\mu}\,\dfrac{1}{\left(1 - \dfrac{\lambda}{c\,\mu}\right)^2} p_0\right] + \dfrac{1}{\mu}\right] \\ O_2\, EkspN\;\; pelanggan \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad \textit{otherwise} \end{array}$$

Jadi ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan *c*:

$$EOT_{MMcGD}(\lambda, \mu, c, O_1, O_2) := \left| \begin{array}{l} c \leftarrow \dfrac{c}{pelayan} \\ \text{if } 0 < \dfrac{\lambda}{c\,\mu} < 1 \\ \quad \left| \begin{array}{l} EOO \leftarrow c\ pelayan\ O_1 \\ EkspON \leftarrow \left| \begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1} \left[\frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \left(\frac{c\,\mu}{c\,\mu - \lambda} \right)} \\ EN \leftarrow \left| \begin{array}{l} ENq \leftarrow \dfrac{1}{c!} \left(\dfrac{\lambda}{\mu} \right)^c \dfrac{\lambda}{c\,\mu} \dfrac{1}{\left(1 - \dfrac{\lambda}{c\,\mu} \right)^2} p_0 \\ ED \leftarrow \dfrac{1}{\lambda_{eff}}\ ENq \\ EW \leftarrow ED + \dfrac{1}{\mu} \\ EN \leftarrow \lambda_{eff}\ EW \end{array} \right. \\ O_2\ EN \end{array} \right. \\ EOO + EkspON\ \ pelanggan \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad otherwise \end{array} \right.$$

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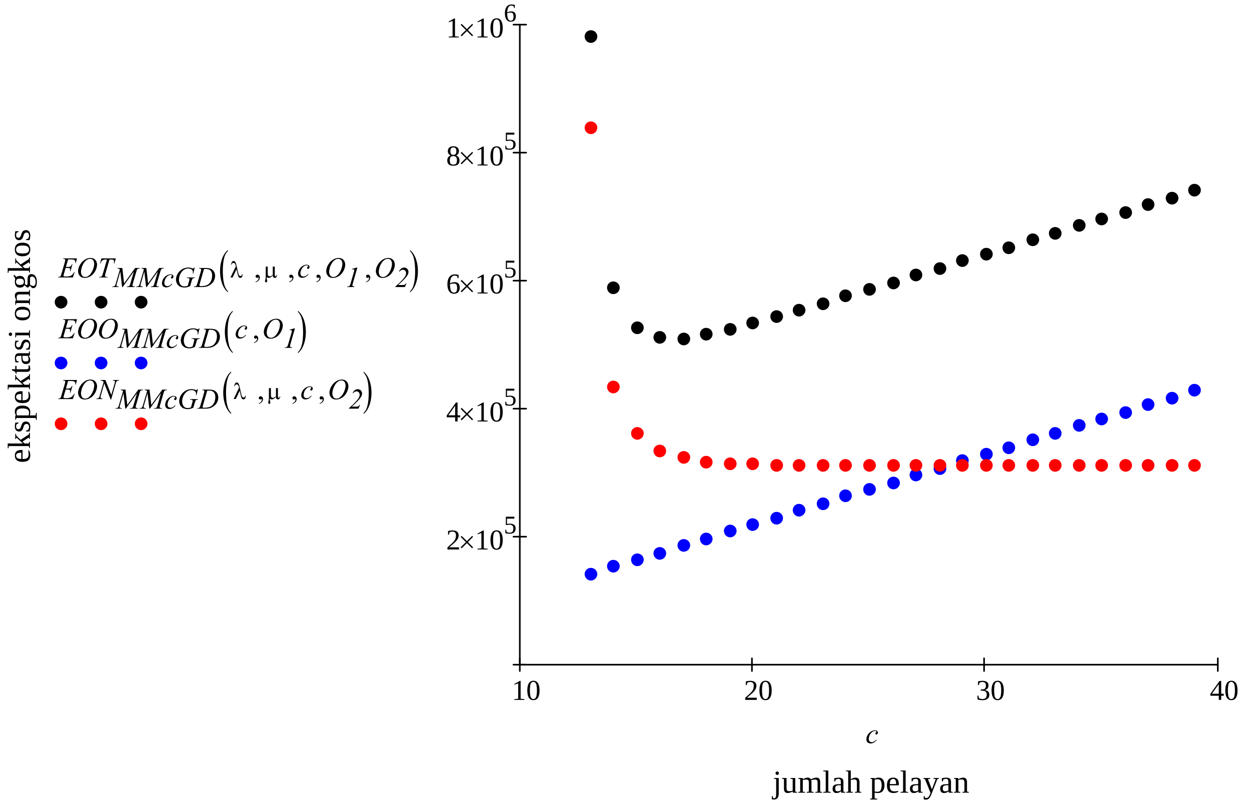

Kurva ekspektasi ongkos vs. jumlah pelayan

$O_1 = 1.1 \times 10^4 \frac{Rp}{jam\ \ pelayan}$ $\qquad$ $O_2 = 2.5 \times 10^4 \frac{Rp}{jam\ \ pelanggan}$

$c =$ (pelayan)	$EOO_{MMcGD}(c, O_1)$ $\frac{Rp}{jam}$	$EON_{MMcGD}(\lambda, \mu, c, O_2)$ $\frac{Rp}{jam}$	$EOT_{MMcGD}(\lambda, \mu, c, O_1, O_2)$ $\frac{Rp}{jam}$
13	$1.43 \cdot 10^5$	$8.407 \cdot 10^5$	$9.837 \cdot 10^5$
14	$1.54 \cdot 10^5$	$4.356 \cdot 10^5$	$5.896 \cdot 10^5$
15	$1.65 \cdot 10^5$	$3.627 \cdot 10^5$	$5.277 \cdot 10^5$
16	$1.76 \cdot 10^5$	$3.361 \cdot 10^5$	$5.121 \cdot 10^5$
17	$1.87 \cdot 10^5$	$3.242 \cdot 10^5$	$5.112 \cdot 10^5$
18	$1.98 \cdot 10^5$	$3.184 \cdot 10^5$	$5.164 \cdot 10^5$
19	$2.09 \cdot 10^5$	$3.155 \cdot 10^5$	$5.245 \cdot 10^5$
20	$2.2 \cdot 10^5$	$3.14 \cdot 10^5$	$5.34 \cdot 10^5$
21	$2.31 \cdot 10^5$	$3.132 \cdot 10^5$	$5.442 \cdot 10^5$
22	$2.42 \cdot 10^5$	$3.128 \cdot 10^5$	$5.548 \cdot 10^5$
23	$2.53 \cdot 10^5$	$3.127 \cdot 10^5$	$5.657 \cdot 10^5$
24	$2.64 \cdot 10^5$	$3.126 \cdot 10^5$	$5.766 \cdot 10^5$
25	$2.75 \cdot 10^5$	$3.125 \cdot 10^5$	$5.875 \cdot 10^5$
...	...	...	...

Ekspektasi jumlah pelanggan dalam sistem:

$$EN_{MMcGD}(\lambda, \mu, c) := \left| \begin{array}{l} c \leftarrow \frac{c}{pelayan} \\ if\ 0 < \frac{\lambda}{c\,\mu} < 1 \\ \quad \left| \begin{array}{l} \lambda_{eff} \leftarrow \lambda \\ p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1} \left[\frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu - \lambda}\right)} \\ EkspN \leftarrow \lambda_{eff} \left[\dfrac{1}{\lambda_{eff}} \left[\dfrac{1}{c!} \left(\dfrac{\lambda}{\mu}\right)^c \dfrac{\lambda}{c\,\mu} \dfrac{1}{\left(1 - \dfrac{\lambda}{c\,\mu}\right)^2} p_0 \right] + \dfrac{1}{\mu} \right] \\ EkspN\ \ pelanggan \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad otherwise \end{array} \right.$$

$c =$		$EN_{MMcGD}(\lambda, \mu, c)$	
13	*pelayan*	33.626	*pelanggan*
14		17.425	
15		14.507	
16		13.443	
17		12.967	
18		12.735	
19		12.618	
20		12.559	
21		12.529	
22		12.514	
23		12.506	
24		12.503	
25		12.501	
...		...	

Jumlah pelayan optimal:

Dalam program ini $c_{atas} \leftarrow 3\ c_{min}$

$$
c_{optMMcGD}(\lambda, \mu, O_1, O_2) :=
\begin{array}{|l}
c_{min} \leftarrow \begin{array}{|ll} ceil\left(\frac{\lambda}{\mu}\right) + 1 & if\ \ ceil\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ ceil\left(\frac{\lambda}{\mu}\right) & otherwise \end{array} \\
c_{atas} \leftarrow 3\ c_{min} \\
for\ \ c \in c_{min} .. c_{atas} \\
\quad v_{EOT_c} \leftarrow \begin{array}{|l} if\ \ 0 < \frac{\lambda}{c\ \mu} < 1 \\
\quad \begin{array}{|l} EOO \leftarrow c\ pelayan\ \ O_1 \\
\lambda_{eff} \leftarrow \lambda \\
p_0 \leftarrow \dfrac{1}{\displaystyle\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\ \mu}{c\ \mu - \lambda}\right)} \\
EkspON \leftarrow \begin{array}{|l} ENq \leftarrow \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda}{c\ \mu} \dfrac{1}{\left(1 - \frac{\lambda}{c\ \mu}\right)^2} p_0 \\
ED \leftarrow \frac{1}{\lambda_{eff}}\ ENq \\
EW \leftarrow ED + \frac{1}{\mu} \\
EN \leftarrow \lambda_{eff}\ EW \\
O_2\ EN \end{array} \\
EOO + EkspON\ \ pelanggan \end{array} \\
\text{"Tidak didefinisikan"}\quad otherwise \end{array} \\
\left(match\left(min\left(v_{EOT}\right), v_{EOT}\right)_{c_{min}}\right)\ pelayan
\end{array}
$$

$c_{optMMcGD}(\lambda, \mu, O_1, O_2) = 17\ pelayan$

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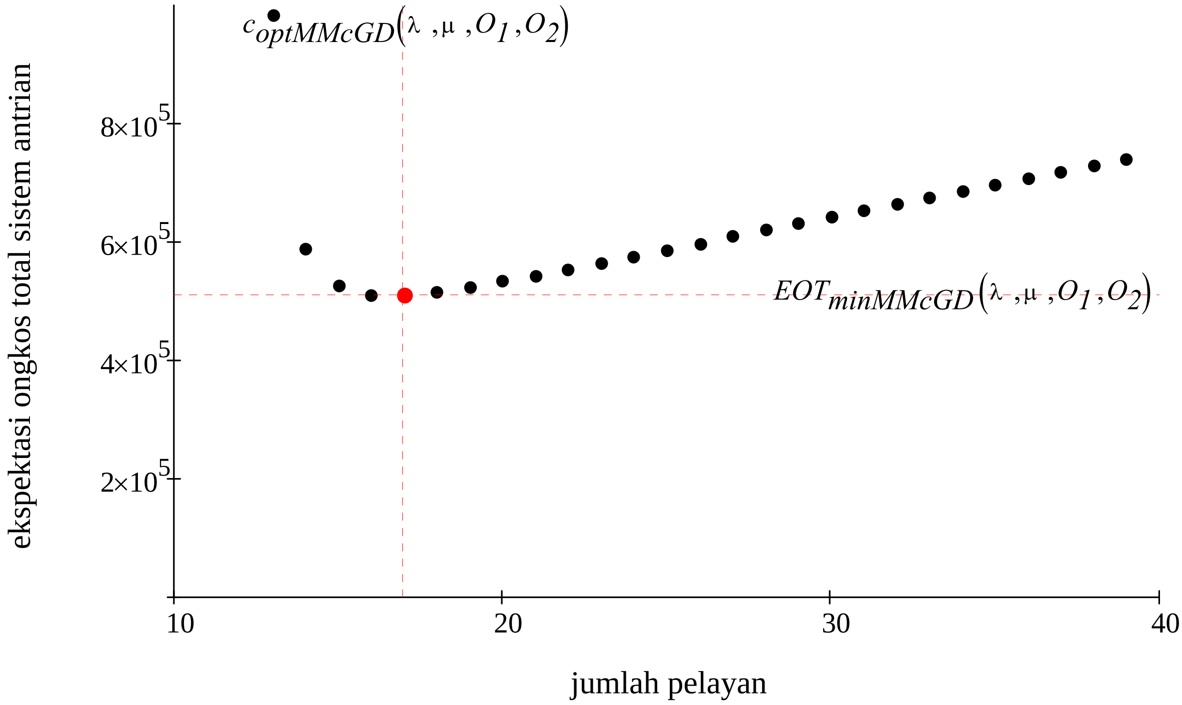
Ekspektasi ongkos total sistem antrian minimum:

Dalam program ini $c_{atas} \leftarrow 3\, c_{min}$

$$
\begin{aligned}
EOT_{minMMcGD}(\lambda, \mu, O_1, O_2) := \; & c_{min} \leftarrow \begin{cases} ceil\left(\frac{\lambda}{\mu}\right) + 1 & \text{if } ceil\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ ceil\left(\frac{\lambda}{\mu}\right) & \text{otherwise} \end{cases} \\
& c_{atas} \leftarrow 3\, c_{min} \\
& \text{for } c \in c_{min} .. c_{atas} \\
& \quad v_{EOT_c} \leftarrow \begin{cases} \left| \begin{aligned} & EOO \leftarrow c\ pelayan\ O_1 \\ & \lambda_{eff} \leftarrow \lambda \\ & p_0 \leftarrow \frac{1}{\sum_{n=0}^{c-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\,\mu}{c\,\mu - \lambda}\right)} \\ & EkspON \leftarrow \left| \begin{aligned} & ENq \leftarrow \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda}{c\,\mu} \frac{1}{\left(1 - \frac{\lambda}{c\,\mu}\right)^2} p_0 \\ & ED \leftarrow \frac{1}{\lambda_{eff}} ENq \\ & EW \leftarrow ED + \frac{1}{\mu} \\ & EN \leftarrow \lambda_{eff}\ EW \\ & O_2\ EN \end{aligned} \right. \\ & EOO + EkspON\ pelanggan \end{aligned} \right. & \text{if } 0 < \frac{\lambda}{c\,\mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases} \\
& EOT_{min} \leftarrow min(v_{EOT})
\end{aligned}
$$

$$EOT_{minMMcGD}(\lambda, \mu, O_1, O_2) = 5.112 \times 10^5\ \frac{Rp}{jam}$$



Kurva ekspektasi ongkos total sistem antrian vs. jumlah pelayan