

## RUMUS-RUMUS SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

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**Sistem antrian dengan waktu antardatang *General*, waktu layan *General*, jumlah pelayan paralel sebesar  $c$ , disiplin antrian *General Discipline*, jumlah pelanggan maksimum yang diperbolehkan dalam sistem sebanyak takhingga, dan jumlah populasi pelanggan takhingga.**

Distribusi umum (*general*) adalah distribusi di mana tidak dibuat perandaian khusus tentang bentuk distribusi probabilitasnya.

$c := 3$  pelayan  $c$  menyatakan jumlah pelayan paralel.

Data mentah variabel acak waktu antardatang  $T$  di kolom ke 0 dan data mentah variabel acak waktu layan  $S$  di kolom ke 1 dari tabel  $Dat$  dalam satuan jam.

$Dat :=$

	0	1
0	0.4	0.2
1	0.2	0.1
2	0.3	0.1
3	0.2	0.3
4	0.2	0.2
5	0.5	0.1

Waktu antar datang  $T$  di tabel  $Dat$  kolom ke 0:

$T := Dat^{(0)}$  jam

$T =$

	0
0	0.4
1	0.2
2	0.3
3	0.2
4	0.2
5	0.5

jam

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Ekspektasi waktu antardatang pelanggan  $ET$  dihitung dari *mean* atau rata-rata variabel acak  $T$ :

$$ET := \text{mean}(T)$$

$$ET = 0.3 \text{ jam}$$

$$T =$$

	0
0	0.4
1	0.2
2	0.3
3	0.2
4	0.2
5	0.5

} *jam*

Momen kedua  $T$  dinotasikan dengan  $ET^2$

$$ET^2 := \frac{\sum_{i=0}^{\text{rows}(T)-1} (T_i)^2}{\text{rows}(T)}$$

$$ET^2 = 0.10333 \text{ jam}^2$$

$$\text{rows}(T) = 6$$

$$T =$$

	0
0	0.4
1	0.2
2	0.3
3	0.2
4	0.2
5	0.5

} *jam*

$$T^2 =$$

	0
0	0.16
1	0.04
2	0.09
3	0.04
4	0.04
5	0.25

} *jam*<sup>2</sup>

Variansi  $T$ :

$$VT := ET^2 - (ET)^2$$

$$VT = 0.01333 \text{ jam}^2$$

$$ET^2 = 0.10333 \text{ jam}^2$$

$$ET = 0.3 \text{ jam}$$

$$(ET)^2 = 0.09 \text{ jam}^2$$

Waktu layan  $S$  di tabel Dat kolom ke 1:

$$S := \text{Dat}^{(1)} \text{ jam}$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \text{ jam} \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array}$$

Ekspektasi waktu layan  $ES$  dihitung dari *mean* atau rata-rata variabel acak  $S$ :

$$ES := \text{mean}(S)$$

$$ES = 0.16667 \text{ jam}$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \text{ jam} \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array}$$

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Momen kedua  $S$  dinotasikan dengan  $ES^2$

$$ES^2 := \frac{\sum_{i=0}^{rows(S)-1} (S_i)^2}{rows(S)}$$

$$ES^2 = 0.033333 \text{ jam}^2$$

$$rows(S) = 6$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array} \text{ jam}$$

$$S^2 = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.04 \\ \hline 1 & 0.01 \\ \hline 2 & 0.01 \\ \hline 3 & 0.09 \\ \hline 4 & 0.04 \\ \hline 5 & 0.01 \\ \hline \end{array} \text{ jam}^2$$

Variansi  $S$ :

$$VS := ES^2 - (ES)^2$$

$$VS = 5.55556 \times 10^{-3} \text{ jam}^2$$

$$ES^2 = 0.033333 \text{ jam}^2$$

$$ES = 0.16667 \text{ jam}$$

$$(ES)^2 = 0.02778 \text{ jam}^2$$

$\lambda$  menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu:

$$\lambda := \frac{1 \text{ pelanggan}}{ET}$$

$$\lambda = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$ET = 0.3 \text{ jam}$$

## RUMUS-RUMUS SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

Laju datang rata-rata efektif:

$$\lambda_{eff} := \lambda$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$\mu$  menyatakan laju layan yaitu jumlah pelanggan yang telah dilayani rata-rata per satuan waktu:

$$\mu := \frac{1 \text{ pelanggan}}{ES}$$

$$\mu = 6 \frac{\text{pelanggan}}{\text{jam}} \quad ES = 0.16667 \text{ jam}$$

$c$  menyatakan jumlah pelayan paralel:

$$c = 3 \text{ pelayan}$$

$\rho$  menyatakan faktor utilisasi, besarnya sama dengan  $\frac{\lambda}{c \mu}$

$$\rho(\lambda, \mu, c) := \frac{\lambda}{\frac{c}{\text{pelayan}} \mu}$$

$$\rho(\lambda, \mu, c) = 0.18519$$

$$\lambda = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$\mu = 6 \frac{\text{pelanggan}}{\text{jam}}$$

$$c = 3 \text{ pelayan}$$

Ekspektasi waktu antri Brumelle yaitu waktu rata-rata pelanggan berada dalam antrian menurut Brumelle:

$$ED_{GGcGD}(\lambda, \mu, c) \geq \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES}$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$ED_{GGcGD}(\lambda, \mu, c) \geq 0 \quad \text{selain itu.}$$

Jadi batas bawah ekspektasi waktu antri Brumelle yaitu batas bawah waktu rata-rata pelanggan berada dalam antrian:

$$ED_{GGcGD\_BB}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\ ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\ ET \leftarrow \frac{ET}{\text{jam}} \\ ES \leftarrow \frac{ES}{\text{jam}} \\ \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ (h1 - h2) \text{ jam} \end{array} \right. \\ 0 \text{ jam} \text{ otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{cases}$$

$$ED_{GGcGD\_BB}(\lambda, \mu, c) = 0 \text{ jam}$$

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$c = 3 \text{ pelayan}$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left( \frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left( \frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \text{ jam}$$

Ekspektasi waktu sistem yaitu waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD}(\lambda, \mu, c) \geq \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} + ES$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$EW_{GGcGD}(\lambda, \mu, c) \geq ES \quad \text{selain itu.}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

Jadi batas bawah ekspektasi waktu sistem yaitu batas bawah waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD\_BB}(\lambda, \mu, c) := \left\{ \begin{array}{l} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\ ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\ ET \leftarrow \frac{ET}{\text{jam}} \\ ES \leftarrow \frac{ES}{\text{jam}} \\ \text{if } \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if } \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ [(h1 - h2) + ES] \text{ jam} \\ (0 + ES) \text{ jam} \text{ otherwise} \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{array} \right. \end{array} \right. \end{array} \right.$$

$$EW_{GGcGD\_BB}(\lambda, \mu, c) = 0.16667 \text{ jam}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left( \frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left( \frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \text{ jam}$$

$$ES = 0.16667 \text{ jam}$$



## RUMUS-RUMUS SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

Ekspektasi jumlah pelanggan sistem:

$$EN_{GGcGD}(\lambda, \mu, c) \geq \lambda_{eff} \left[ \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} + ES \right]$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$EN_{GGcGD}(\lambda, \mu, c) \geq \lambda_{eff} ES \quad \text{selain itu.}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

Jadi batas bawah ekspektasi jumlah pelanggan sistem:

$$\begin{aligned}
 EN_{GGcGD\_BB}(\lambda, \mu, c) := & \quad c \leftarrow \frac{c}{\text{pelayan}} \\
 & \quad \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\
 & \quad ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\
 & \quad ET \leftarrow \frac{ET}{\text{jam}} \\
 & \quad ES \leftarrow \frac{ES}{\text{jam}} \\
 & \quad \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\
 & \quad \left| \begin{array}{l}
 \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\
 \quad \left| \begin{array}{l}
 h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\
 h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\
 \lambda_{eff} [(h1 - h2) + ES] \text{ pelanggan} \\
 \lambda_{eff} (0 + ES) \text{ pelanggan otherwise}
 \end{array} \right. \\
 \text{"Tidak didefinisikan" otherwise}
 \end{array} \right.
 \end{aligned}$$

$$EN_{GGcGD\_BB}(\lambda, \mu, c) = 0.55556 \text{pelanggan}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left( \frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left( \frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \text{jam}$$

## RUMUS-RUMUS SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

Ekspektasi jumlah pelanggan antri:

$$EN_{qGGcGD}(\lambda, \mu, c) \geq \lambda_{eff} \left[ \frac{ES^2 - 2cETES}{2c(cET - ES)} - \frac{(c-1)ES^2}{2cES} \right]$$

jika

$$0 < \frac{\lambda}{c\mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2cETES}{cET - ES} \geq \frac{(c-1)ES^2}{ES}$$

$$EN_{qGGcGD}(\lambda, \mu, c) \geq 0 \quad \text{selain itu.}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

Jadi batas bawah ekspektasi jumlah pelanggan antri:

$$\begin{aligned}
 EN_{qGGcGD\_BB}(\lambda, \mu, c) := & \quad c \leftarrow \frac{c}{\text{pelayan}} \\
 & \quad \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\
 & \quad ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\
 & \quad ET \leftarrow \frac{ET}{\text{jam}} \\
 & \quad ES \leftarrow \frac{ES}{\text{jam}} \\
 & \quad \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\
 & \quad \quad \left| \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \right. \\
 & \quad \quad \quad \left| h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \right. \\
 & \quad \quad \quad \quad \left| h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \right. \\
 & \quad \quad \quad \quad \quad \left| \lambda_{eff} (h1 - h2) \text{ pelanggan} \right. \\
 & \quad \quad \quad \quad \quad \left| \lambda_{eff} (0) \text{ pelanggan otherwise} \right. \\
 & \quad \quad \quad \quad \quad \quad \text{"Tidak didefinisikan" otherwise}
 \end{aligned}$$

$$EN_{qGGcGD\_BB}(\lambda, \mu, c) = 0 \text{ pelanggan}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left( \frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left( \frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \text{ jam}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

**Ekspektasi waktu antri yaitu waktu rata-rata pelanggan berada dalam antrian menurut Brumelle-Kingman:**

$$ED_{GGcGD}(\lambda, \mu, c) \leq \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \quad \text{jika}$$

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$ED_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

**Batas atas ekspektasi waktu antri menurut Brumelle-Kingman:**

$$ED_{GGcGD\_BA}(\lambda, \mu, c) := \left. \begin{array}{l} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\ ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\ ET \leftarrow \frac{ET}{\text{jam}} \\ ES \leftarrow \frac{ES}{\text{jam}} \\ VT \leftarrow \frac{VT}{\text{jam}^2} \\ VS \leftarrow \frac{VS}{\text{jam}^2} \\ \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ h1 \text{ jam} \\ \infty \text{ jam otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{array} \right. \end{array} \right.$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

$$ED_{GGcGD\_BA}(\lambda, \mu, c) = 0.04369 \text{ jam}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

$$\frac{\left(\frac{c}{\text{pelayan}}\right)^2 VT + VS + \left(\frac{c}{\text{pelayan}} - 1\right) ES^2}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES\right)} = 0.04369 \text{ jam}$$

**Ekspektasi waktu sistem menurut Brumelle-Kingman:**

$$EW_{GGcGD}(\lambda, \mu, c) \leq \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} + ES \quad \text{jika}$$

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$ED_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

**Batas atas ekspektasi waktu sistem menurut Brumelle-Kingman:**

$$\begin{aligned}
 EW_{GGcGD\_BA}(\lambda, \mu, c) := & \quad c \leftarrow \frac{c}{\text{pelayan}} \\
 & \quad \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\
 & \quad ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\
 & \quad ET \leftarrow \frac{ET}{\text{jam}} \\
 & \quad ES \leftarrow \frac{ES}{\text{jam}} \\
 & \quad VT \leftarrow \frac{VT}{\text{jam}^2} \\
 & \quad VS \leftarrow \frac{VS}{\text{jam}^2} \\
 & \quad \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\
 & \quad \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \quad \left| \begin{array}{l} h1 \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ (h1 + ES) \text{ jam} \end{array} \right. \\ \quad \infty \text{ jam otherwise} \end{array} \right. \\
 & \quad \text{"Tidak didefinisikan" otherwise}
 \end{aligned}$$

$$EW_{GGcGD\_BA}(\lambda, \mu, c) = 0.21035 \text{ jam}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

## RUMUS-RUMUS SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

$$\frac{\left(\frac{c}{\text{pelayan}}\right)^2 VT + VS + \left(\frac{c}{\text{pelayan}} - 1\right) ES^2}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES\right)} = 0.04369 \text{ jam}$$

$$ES = 0.16667 \text{ jam}$$

**Ekspektasi jumlah pelanggan sistem menurut Brumelle-Kingman:**

$$EN_{GGcGD}(\lambda, \mu, c) \leq \lambda_{\text{eff}} \left[ \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} + ES \right]$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$EN_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$



**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

**Batas atas ekspektasi jumlah pelanggan sistem menurut Brumelle-Kingman:**

$$\begin{aligned}
 EN_{GGcGD\_BA}(\lambda, \mu, c) := & \left\{ \begin{array}{l}
 c \leftarrow \frac{c}{\text{pelayan}} \\
 \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\
 ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\
 ET \leftarrow \frac{ET}{\text{jam}} \\
 ES \leftarrow \frac{ES}{\text{jam}} \\
 VT \leftarrow \frac{VT}{\text{jam}^2} \\
 VS \leftarrow \frac{VS}{\text{jam}^2} \\
 \text{if } \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\
 \left| \begin{array}{l}
 \text{if } \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\
 \left| \begin{array}{l}
 h1 \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\
 \lambda_{eff} (h1 + ES) \text{ pelanggan} \\
 \infty \text{ pelanggan otherwise}
 \end{array} \right. \\
 \text{"Tidak didefinisikan" otherwise}
 \end{array} \right.
 \end{array} \right.
 \end{aligned}$$

$$EN_{GGcGD\_BA}(\lambda, \mu, c) = 0.70118 \text{pelanggan}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

$$\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} = 0.04369 \text{ jam}$$

$$ES = 0.16667 \text{ jam}$$

$$\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} + ES = 0.21035 \text{ jam}$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$\lambda_{eff} \left[ \frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} + ES \right] = 0.70118 \text{ pelanggan}$$

**Ekspektasi jumlah pelanggan antri menurut Brumelle-Kingman:**

$$EN_{qGGcGD}(\lambda, \mu, c) \leq \lambda_{eff} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)}$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$EN_{qGGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

**Batas atas ekspektasi jumlah pelanggan antri menurut Brumelle-Kingman:**

$$\begin{aligned}
 EN_{qGGcGD\_BA}(\lambda, \mu, c) := & \quad c \leftarrow \frac{c}{\text{pelayan}} \\
 & \quad \lambda_{eff} \leftarrow \lambda \frac{\text{jam}}{\text{pelanggan}} \\
 & \quad ES^2 \leftarrow \frac{ES^2}{\text{jam}^2} \\
 & \quad ET \leftarrow \frac{ET}{\text{jam}} \\
 & \quad ES \leftarrow \frac{ES}{\text{jam}} \\
 & \quad VT \leftarrow \frac{VT}{\text{jam}^2} \\
 & \quad VS \leftarrow \frac{VS}{\text{jam}^2} \\
 & \quad \text{if} \left( 0 < \frac{\lambda}{c \mu} < 1 \right) \\
 & \quad \quad \left| \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \right. \\
 & \quad \quad \quad \left| h1 \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \right. \\
 & \quad \quad \quad \quad \left| \lambda_{eff} \quad h1 \text{ pelanggan} \right. \\
 & \quad \quad \quad \quad \left| \infty \text{ pelanggan otherwise} \right. \\
 & \quad \quad \quad \quad \left| \text{"Tidak didefinisikan"} \text{ otherwise} \right.
 \end{aligned}$$

$$EN_{qGGcGD\_BA}(\lambda, \mu, c) = 0.14562 \text{ pelanggan}$$

$$\frac{\lambda}{\frac{c}{\text{pelayan}} \mu} = 0.18519$$

**RUMUS-RUMUS SISTEM ANTRIAN  $G/G/c/GD/\infty/\infty$**

$$\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} = 0.04369 \text{ jam}$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$\lambda_{eff} \left[ \frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} \right] = 0.14562 \text{ pelanggan}$$

**Median ekspektasi waktu antri yaitu median waktu rata-rata pelanggan berada dalam antrian besarnya sama dengan (batas bawahnya + batas atasnya)/2:**

$$ED_{GGcGD\_Median}(\lambda, \mu, c) := \frac{ED_{GGcGD\_BA}(\lambda, \mu, c) + ED_{GGcGD\_BB}(\lambda, \mu, c)}{2}$$

$$ED_{GGcGD\_Median}(\lambda, \mu, c) = 0.02184 \text{ jam}$$

$$ED_{GGcGD\_BB}(\lambda, \mu, c) = 0 \text{ jam}$$

$$ED_{GGcGD\_BA}(\lambda, \mu, c) = 0.04369 \text{ jam}$$

**Median ekspektasi waktu sistem yaitu median waktu rata-rata pelanggan berada dalam sistem besarnya (batas bawahnya + batas atasnya)/2:**

$$EW_{GGcGD\_Median}(\lambda, \mu, c) := \frac{EW_{GGcGD\_BB}(\lambda, \mu, c) + EW_{GGcGD\_BA}(\lambda, \mu, c)}{2}$$

$$EW_{GGcGD\_Median}(\lambda, \mu, c) = 0.18851 \text{ jam}$$

$$EW_{GGcGD\_BB}(\lambda, \mu, c) = 0.16667 \text{ jam}$$

$$EW_{GGcGD\_BA}(\lambda, \mu, c) = 0.21035 \text{ jam}$$

**Median ekspektasi pelanggan sistem yaitu median rata-rata pelanggan dalam sistem besarnya (batas bawahnya + batas atasnya)/2:**

$$EN_{GGcGD\_Median}(\lambda, \mu, c) := \frac{EN_{GGcGD\_BB}(\lambda, \mu, c) + EN_{GGcGD\_BA}(\lambda, \mu, c)}{2}$$

$$EN_{GGcGD\_Median}(\lambda, \mu, c) = 0.62837 \text{ pelanggan}$$

$$EN_{GGcGD\_BB}(\lambda, \mu, c) = 0.55556 \text{ pelanggan}$$

$$EN_{GGcGD\_BA}(\lambda, \mu, c) = 0.70118 \text{ pelanggan}$$

**Median ekspektasi pelanggan antri yaitu median rata-rata pelanggan dalam antrian besarnya:**

$$EN_{qGGcGD\_Median}(\lambda, \mu, c) := \frac{EN_{qGGcGD\_BB}(\lambda, \mu, c) + EN_{qGGcGD\_BA}(\lambda, \mu, c)}{2}$$

$$EN_{qGGcGD\_Median}(\lambda, \mu, c) = 0.07281 \text{ pelanggan}$$

$$\frac{EN_{qGGcGD\_BB}(\lambda, \mu, c) + EN_{qGGcGD\_BA}(\lambda, \mu, c)}{2} = 0.07281 \text{ pelanggan}$$

$$EN_{qGGcGD\_BB}(\lambda, \mu, c) = 0 \text{ pelanggan}$$

$$EN_{qGGcGD\_BA}(\lambda, \mu, c) = 0.14562 \text{ pelanggan}$$