

SISTEM ANTRIAN $G/G/c/GD/\infty/\infty$

Oleh: **Dr. Ir. H. Muhammad Sutarno, S.H.I., M.Sc., M.Ag.**

Sistem antrian dengan waktu antardatang *General*, waktu layan *General*, jumlah pelayan paralel sebesar c , disiplin antrian *General Discipline*, jumlah pelanggan maksimum yang diperbolehkan dalam sistem sebanyak takhingga, dan jumlah populasi pelanggan takhingga.

$c := 3$ pelayan c menyatakan jumlah pelayan paralel.

Data mentah variabel acak waktu antar datang T di kolom ke 0 dan data mentah variabel acak waktu layan S di kolom ke 1 dari tabel *Dat* dalam satuan jam/pelanggan.

Dat :=

| | 0 | 1 |
|---|-----|-----|
| 0 | 0.4 | 0.2 |
| 1 | 0.2 | 0.1 |
| 2 | 0.3 | 0.1 |
| 3 | 0.2 | 0.3 |
| 4 | 0.2 | 0.2 |
| 5 | 0.5 | 0.1 |

Waktu antar datang T di tabel *Dat* kolom ke 0:

$$T := \text{Dat}^{(0)} \frac{\text{jam}}{\text{pelanggan}}$$

$$T = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.4 \\ \hline 1 & 0.2 \\ \hline 2 & 0.3 \\ \hline 3 & 0.2 \\ \hline 4 & 0.2 \\ \hline 5 & 0.5 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu antardatang pelanggan ET dihitung dari *mean* atau rata-rata variabel acak T :

$$ET := \text{mean}(T)$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

$$T = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.4 \\ \hline 1 & 0.2 \\ \hline 2 & 0.3 \\ \hline 3 & 0.2 \\ \hline 4 & 0.2 \\ \hline 5 & 0.5 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

Momen kedua T dinotasikan dengan ET^2 :

$$ET^2 := \frac{\sum_{i=0}^{\text{rows}(T)-1} (T_i)^2}{\text{rows}(T)}$$

$$ET^2 = 0.10333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$\text{rows}(T) = 6$$

$$T = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.4 \\ \hline 1 & 0.2 \\ \hline 2 & 0.3 \\ \hline 3 & 0.2 \\ \hline 4 & 0.2 \\ \hline 5 & 0.5 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

$$T^2 = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.16 \\ \hline 1 & 0.04 \\ \hline 2 & 0.09 \\ \hline 3 & 0.04 \\ \hline 4 & 0.04 \\ \hline 5 & 0.25 \\ \hline \end{array} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Variansi T :

$$VT := ET^2 - (ET)^2$$

$$VT = 0.01333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ET^2 = 0.10333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

$$(ET)^2 = 0.09 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Waktu layan S di tabel Dat kolom ke 1:

$$S := Dat^{(1)} \frac{jam}{pelanggan}$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array} \frac{jam}{pelanggan}$$

Ekspektasi waktu layan ES dihitung dari *mean* atau rata-rata variabel acak S :

$$ES := mean(S)$$

$$ES = 0.16667 \frac{jam}{pelanggan}$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array} \frac{jam}{pelanggan}$$

Momen kedua S dinotasikan dengan ES^2 :

$$ES^2 := \frac{\sum_{i=0}^{rows(S)-1} (S_i)^2}{rows(S)}$$

$$ES^2 = 0.03333 \frac{jam^2}{pelanggan^2}$$

$$rows(S) = 6$$

$$S = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.2 \\ \hline 1 & 0.1 \\ \hline 2 & 0.1 \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.1 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

$$S^2 = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & 0.04 \\ \hline 1 & 0.01 \\ \hline 2 & 0.01 \\ \hline 3 & 0.09 \\ \hline 4 & 0.04 \\ \hline 5 & 0.01 \\ \hline \end{array} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Variansi S:

$$VS := ES^2 - (ES)^2$$

$$VS = 5.55556 \times 10^{-3} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ES^2 = 0.03333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

$$(ES)^2 = 0.02778 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu:

$$\lambda := \frac{1}{ET}$$

$$\lambda = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

Laju datang rata-rata efektif:

$$\lambda_{eff} := \lambda$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

μ menyatakan laju layan yaitu jumlah pelanggan yang telah dilayani rata-rata per satuan waktu:

$$\mu := \frac{1}{ES}$$

$$\mu = 6 \frac{\text{pelanggan}}{\text{jam}}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

c menyatakan jumlah pelayan paralel:

$$c = 3 \text{ pelayan}$$

ρ menyatakan faktor utilisasi, besarnya sama dengan $\frac{\lambda}{c \mu}$

$$\rho(\lambda, \mu, c) := \frac{\lambda}{c \mu}$$

$$\rho(\lambda, \mu, c) = 0.18519 \frac{1}{\text{pelayan}}$$

$$\lambda = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$\mu = 6 \frac{\text{pelanggan}}{\text{jam}}$$

$$c = 3 \text{ pelayan}$$

Ekspektasi waktu antri yaitu waktu rata-rata pelanggan berada dalam antrian:

$$ED_{GGcGD}(\lambda, \mu, c) \geq \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES}$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$ED_{GGcGD}(\lambda, \mu, c) \geq 0 \quad \text{selain itu.}$$

Jadi batas bawah ekspektasi waktu antri yaitu batas bawah waktu rata-rata pelanggan berada dalam antrian:

$$ED_{GGcGD_BB}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ h1 - h2 \\ 0 \frac{\text{jam}}{\text{pelanggan}} \text{ otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{array} \right. \end{cases}$$

$$ED_{GGcGD_BB}(\lambda, \mu, c) = 0 \frac{\text{jam}}{\text{pelanggan}}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{I}{\text{pelayan}}$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu sistem yaitu waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD}(\lambda, \mu, c) \geq \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} + ES$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$EW_{GGcGD}(\lambda, \mu, c) \geq ES \quad \text{selain itu.}$$

Jadi batas bawah ekspektasi waktu sistem yaitu batas bawah waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD_BB}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} (c ET - ES)} - \frac{(c-1) ES^2}{2 c ES} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} (c ET - ES)} \\ h2 \leftarrow \frac{(c-1) ES^2}{2 c ES} \\ (h1 - h2) + ES \end{array} \right. \\ \left(0 \frac{\text{jam}}{\text{pelanggan}} + ES \right) \text{ otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{cases}$$

$$EW_{GGcGD_BB}(\lambda, \mu, c) = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{1}{\text{pelayan}}$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \frac{\text{jam}}{\text{pelanggan}}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi jumlah pelanggan dalam sistem:

$$EN_{GGcGD}(\lambda, \mu, c) \geq \lambda_{eff} \left[\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} (c ET - ES)} - \frac{(c-1) ES^2}{2 c ES} + ES \right]$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$EN_{GGcGD}(\lambda, \mu, c) \geq \lambda_{eff} ES \quad \text{selain itu.}$$

Jadi batas bawah ekspektasi jumlah pelanggan dalam sistem:

$$EN_{GGcGD_BB}(\lambda, \mu, c) := \left\{ \begin{array}{l} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left\{ \begin{array}{l} \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \left\{ \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ \lambda_{eff} [(h1 - h2) + ES] \text{ pelanggan} \\ \lambda_{eff} (0 + ES) \text{ pelanggan otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{array} \right. \end{array} \right.$$

$$EN_{GGcGD_BB}(\lambda, \mu, c) = 0.55556 \text{ pelanggan}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{1}{\text{pelayan}}$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi jumlah pelanggan antri:

$$EN_{qGGcGD}(\lambda, \mu, c) \geq \lambda_{eff} \left[\frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \right]$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{ES^2 - 2 c ET ES}{c ET - ES} \geq \frac{(c - 1) ES^2}{ES}$$

$$EN_{qGGcGD}(\lambda, \mu, c) \geq 0 \quad \text{selain itu.}$$

Jadi batas bawah ekspektasi jumlah pelanggan antri:

$$EN_{qGGcGD_BB}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \quad \left| \begin{array}{l} \text{if} \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \quad \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ \lambda_{eff} (h1 - h2) \text{ pelanggan} \\ \lambda_{eff} (0) \text{ pelanggan otherwise} \end{array} \right. \\ \lambda_{eff} (0) \text{ pelanggan otherwise} \\ \text{"Tidak didefinisikan" otherwise} \end{array} \right. \end{cases}$$

$$EN_{qGGcGD_BB}(\lambda, \mu, c) = 0 \text{ pelanggan}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{1}{\text{pelayan}}$$

$$\frac{ES^2 - 2 \frac{c}{\text{pelayan}} ET ES}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} - \frac{\left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} ES} = -0.12727 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu antri yaitu waktu rata-rata pelanggan berada dalam antrian:

$$ED_{GGcGD}(\lambda, \mu, c) \leq \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)}$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$ED_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

Batas atas ekspektasi waktu antri yaitu batas atas waktu rata-rata pelanggan antri:

$$ED_{GGcGD_BA}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{\text{eff}} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \quad \text{if} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \infty \frac{\text{jam}}{\text{pelanggan}} \quad \text{otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan"} \quad \text{otherwise} \end{cases}$$

$$ED_{GGcGD_BA}(\lambda, \mu, c) = 0.04369 \frac{\text{jam}}{\text{pelanggan}}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{I}{\text{pelayan}}$$

$$\frac{\left(\frac{c}{\text{pelayan}} \right)^2 VT + VS + \left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} = 0.04369 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu sistem yaitu waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD}(\lambda, \mu, c) \leq \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} + ES$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$ED_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

Batas atas ekspektasi waktu sistem yaitu batas atas waktu rata-rata pelanggan berada dalam sistem:

$$EW_{GGcGD_BA}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \left| \begin{array}{l} h \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ h + ES \end{array} \right. \\ \infty \frac{\text{jam}}{\text{pelanggan}} \text{ otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{cases}$$

$$EW_{GGcGD_BA}(\lambda, \mu, c) = 0.21035 \frac{\text{jam}}{\text{pelanggan}}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{I}{\text{pelayan}}$$

$$\frac{\left(\frac{c}{\text{pelayan}} \right)^2 VT + VS + \left(\frac{c}{\text{pelayan}} - 1 \right) ES^2}{2 \frac{c}{\text{pelayan}} \left(\frac{c}{\text{pelayan}} ET - ES \right)} = 0.04369 \frac{\text{jam}}{\text{pelanggan}}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi pelanggan sistem yaitu rata-rata jumlah pelanggan berada dalam sistem:

$$EN_{GGcGD}(\lambda, \mu, c) \leq \lambda_{eff} \left[\frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} + ES \right]$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$EN_{GGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

Batas atas ekspektasi jumlah pelanggan dalam sistem:

$$EN_{GGcGD_BA}(\lambda, \mu, c) := \left\{ \begin{array}{l} c \leftarrow \frac{c}{pelayan} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \left| \begin{array}{l} h \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ \lambda_{eff} (h + ES) \text{ pelanggan} \end{array} \right. \\ \lambda_{eff} \left(\infty \frac{\text{jam}}{\text{pelanggan}} \right) \text{ pelanggan otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{array} \right.$$

$$EN_{GGcGD_BA}(\lambda, \mu, c) = 0.70118 \text{ pelanggan}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{I}{pelayan}$$

$$\frac{\left(\frac{c}{pelayan} \right)^2 VT + VS + \left(\frac{c}{pelayan} - 1 \right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES \right)} = 0.04369 \frac{\text{jam}}{\text{pelanggan}}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

$$\frac{\left(\frac{c}{pelayan} \right)^2 VT + VS + \left(\frac{c}{pelayan} - 1 \right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES \right)} + ES = 0.21035 \frac{\text{jam}}{\text{pelanggan}}$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$\lambda_{eff} \left[\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} + ES \right] pelanggan = 0.70118 \text{ pelanggan}$$

Ekspektasi jumlah pelanggan antri yaitu rata-rata pelanggan berada dalam antrian:

$$EN_{qGGcGD}(\lambda, \mu, c) \leq \lambda_{eff} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)}$$

jika

$$0 < \frac{\lambda}{c \mu} < 1 \quad \text{dan} \quad \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0$$

$$EN_{qGGcGD}(\lambda, \mu, c) \leq \infty \quad \text{selain itu.}$$

Batas atas ekspektasi jumlah pelanggan antri:

$$EN_{qGGcGD_BA}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{pelayan} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \left| \begin{array}{l} h \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ \lambda_{eff} (h) \text{ pelanggan} \end{array} \right. \\ \lambda_{eff} \left(\infty \frac{\text{jam}}{\text{pelanggan}} \right) \text{ pelanggan otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{cases}$$

$$EN_{qGGcGD_BA}(\lambda, \mu, c) = 0.14562 \text{ pelanggan}$$

$$\frac{\lambda}{c \mu} = 0.18519 \frac{1}{pelayan}$$

$$\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} = 0.04369 \frac{jam}{pelanggan}$$

$$\lambda_{eff} = 3.33333 \frac{pelanggan}{jam}$$

$$\lambda_{eff} \left[\frac{\left(\frac{c}{pelayan}\right)^2 VT + VS + \left(\frac{c}{pelayan} - 1\right) ES^2}{2 \frac{c}{pelayan} \left(\frac{c}{pelayan} ET - ES\right)} \right] pelanggan = 0.14562 \text{ pelanggan}$$

Median ekspektasi waktu antri yaitu median waktu rata-rata pelanggan berada dalam antrian besarnya sama dengan (batas bawahnya + batas atasnya)/2:

$$ED_{GGcGD_Median}(\lambda, \mu, c) := \frac{ED_{GGcGD_BA}(\lambda, \mu, c) + ED_{GGcGD_BB}(\lambda, \mu, c)}{2}$$

$$ED_{GGcGD_Median}(\lambda, \mu, c) = 0.02184 \frac{jam}{pelanggan}$$

$$ED_{GGcGD_BB}(\lambda, \mu, c) = 0 \frac{jam}{pelanggan}$$

$$ED_{GGcGD_BA}(\lambda, \mu, c) = 0.04369 \frac{jam}{pelanggan}$$

Median ekspektasi waktu sistem yaitu median waktu rata-rata pelanggan berada dalam sistem besarnya (batas bawahnya + batas atasnya)/2:

$$EW_{GGcGD_Median}(\lambda, \mu, c) := \frac{EW_{GGcGD_BB}(\lambda, \mu, c) + EW_{GGcGD_BA}(\lambda, \mu, c)}{2}$$

$$EW_{GGcGD_Median}(\lambda, \mu, c) = 0.18851 \frac{jam}{pelanggan}$$

$$EW_{GGcGD_BB}(\lambda, \mu, c) = 0.16667 \frac{jam}{pelanggan}$$

$$EW_{GGcGD_BA}(\lambda, \mu, c) = 0.21035 \frac{jam}{pelanggan}$$

Median ekspektasi pelanggan sistem yaitu median rata-rata pelanggan dalam sistem besarnya (batas bawahnya + batas atasnya)/2:

$$EN_{GGcGD_Median}(\lambda, \mu, c) := \frac{EN_{GGcGD_BB}(\lambda, \mu, c) + EN_{GGcGD_BA}(\lambda, \mu, c)}{2}$$

$$EN_{GGcGD_Median}(\lambda, \mu, c) = 0.62837 \text{ pelanggan}$$

$$EN_{GGcGD_BB}(\lambda, \mu, c) = 0.55556 \text{ pelanggan}$$

$$EN_{GGcGD_BA}(\lambda, \mu, c) = 0.70118 \text{ pelanggan}$$

Median ekspektasi pelanggan antri yaitu median rata-rata pelanggan dalam antrian besarnya:

$$EN_{qGGcGD_Median}(\lambda, \mu, c) := \begin{cases} h \leftarrow \frac{EN_{GGcGD_BB}(\lambda, \mu, c) + EN_{GGcGD_BA}(\lambda, \mu, c)}{2} \\ h - \lambda_{eff} \text{ ES pelanggan} \end{cases}$$

$$EN_{qGGcGD_Median}(\lambda, \mu, c) = 0.07281 \text{ pelanggan}$$

$$\frac{EN_{GGcGD_BB}(\lambda, \mu, c) + EN_{GGcGD_BA}(\lambda, \mu, c)}{2} = 0.62837 \text{ pelanggan}$$

$$EN_{GGcGD_BB}(\lambda, \mu, c) = 0.55556 \text{ pelanggan}$$

$$EN_{GGcGD_BA}(\lambda, \mu, c) = 0.70118 \text{ pelanggan}$$

$$\lambda_{eff} \text{ ES pelanggan} = 0.55556 \text{ pelanggan}$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$