

MODEL KEPUTUSAN ANTRIAN $G/G/c/GD/\infty/\infty$ MODEL ONGKOS

JUMLAH PELAYAN OPTIMAL

Oleh: Dr. Ir. H. Muhammad Sutarno, S.H.I., M.Sc., M.Ag.

Sistem antrian dengan waktu antardatang *General*, waktu layan *General*, jumlah pelayan paralel sebesar c , disiplin antrian *General Discipline*, jumlah pelanggan maksimum yang diperbolehkan dalam sistem sebanyak takhingga, dan jumlah populasi pelanggan takhingga.

Data mentah variabel acak waktu antar datang T dan data mentah variabel acak waktu layan S dalam satuan jam/pelanggan:

$$T \equiv \begin{pmatrix} 0.4 \\ 0.2 \\ 0.3 \\ 0.2 \\ 0.2 \\ 0.5 \end{pmatrix} \frac{\text{jam}}{\text{pelanggan}} \qquad S \equiv \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu antardatang pelanggan ET dihitung dari *mean* atau rata-rata variabel acak T :

$$ET \equiv \text{mean}(T)$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

Momen kedua T dinotasikan dengan ET^2 :

$$ET^2 \equiv \frac{\sum_{i = ORIGIN}^{rows(T)+ORIGIN-1} (T_i)^2}{rows(T)} \qquad ORIGIN = 1$$

$$ET^2 = 0.10333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$\text{rows}(T) = 6$$

$$\text{ORIGIN} = 1$$

$$T = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.4 \\ \hline 2 & 0.2 \\ \hline 3 & 0.3 \\ \hline 4 & 0.2 \\ \hline 5 & 0.2 \\ \hline 6 & 0.5 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

$$T^2 = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.16 \\ \hline 2 & 0.04 \\ \hline 3 & 0.09 \\ \hline 4 & 0.04 \\ \hline 5 & 0.04 \\ \hline 6 & 0.25 \\ \hline \end{array} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$\sum_{i = \text{ORIGIN}}^{\text{rows}(T)+\text{ORIGIN}-1} (T_i)^2 = 0.62 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Variansi T :

$$VT \equiv ET^2 - (ET)^2$$

$$VT = 0.01333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ET^2 = 0.10333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

$$(ET)^2 = 0.09 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Waktu layan S :

$$S = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.2 \\ \hline 2 & 0.1 \\ \hline 3 & 0.1 \\ \hline 4 & 0.3 \\ \hline 5 & 0.2 \\ \hline 6 & 0.1 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

Ekspektasi waktu layan ES dihitung dari *mean* atau rata-rata variabel acak S :

$$ES \equiv \text{mean}(S)$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

$$S = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.2 \\ \hline 2 & 0.1 \\ \hline 3 & 0.1 \\ \hline 4 & 0.3 \\ \hline 5 & 0.2 \\ \hline 6 & 0.1 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

Momen kedua S dinotasikan dengan ES^2 :

$$ES^2 \equiv \frac{\sum_{i=ORIGIN}^{rows(S)+ORIGIN-1} (S_i)^2}{rows(S)}$$

$$ES^2 = 0.03333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$rows(S) = 6 \qquad ORIGIN = 1$$

$$S = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.2 \\ \hline 2 & 0.1 \\ \hline 3 & 0.1 \\ \hline 4 & 0.3 \\ \hline 5 & 0.2 \\ \hline 6 & 0.1 \\ \hline \end{array} \frac{\text{jam}}{\text{pelanggan}}$$

$$S^2 = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 0.04 \\ \hline 2 & 0.01 \\ \hline 3 & 0.01 \\ \hline 4 & 0.09 \\ \hline 5 & 0.04 \\ \hline 6 & 0.01 \\ \hline \end{array} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

Variansi S :

$$VS := ES^2 - (ES)^2$$

$$VS = 5.55556 \times 10^{-3} \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ES^2 = 0.033333 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

$$ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}} \quad (ES)^2 = 0.02778 \frac{\text{jam}^2}{\text{pelanggan}^2}$$

λ menyatakan laju datang (*arrival rate*) yaitu jumlah pelanggan yang datang rata-rata per satuan waktu:

$$\lambda \equiv \frac{1}{ET}$$

$$\lambda = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

$$ET = 0.3 \frac{\text{jam}}{\text{pelanggan}}$$

Laju datang rata-rata efektif:

$$\lambda_{eff} \equiv \lambda$$

$$\lambda_{eff} = 3.33333 \frac{\text{pelanggan}}{\text{jam}}$$

μ menyatakan laju layan yaitu jumlah pelanggan yang telah dilayani rata-rata per satuan waktu:

$$\mu \equiv \frac{1}{ES}$$

$$\mu = 6 \frac{\text{pelanggan}}{\text{jam}} \quad ES = 0.16667 \frac{\text{jam}}{\text{pelanggan}}$$

Ongkos pelayanan per pelayan per satuan waktu:

$$O_1 \equiv 30000 \frac{\text{Rp}}{\text{jam pelayan}}$$

Ongkos (*nilai*) per pelanggan dalam sistem per satuan waktu:

$$O_2 \equiv 3000000 \frac{Rp}{jam \text{ pelanggan}}$$

Jumlah pelayan minimum:

$$c_{min}(\lambda, \mu) \equiv \begin{cases} \left(\text{ceil}\left(\frac{\lambda}{\mu}\right) + 1 \right) \text{ pelayan} & \text{if } \text{ceil}\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ \left(\text{ceil}\left(\frac{\lambda}{\mu}\right) \right) \text{ pelayan} & \text{otherwise} \end{cases}$$

$$c_{min}(\lambda, \mu) = 1 \text{ pelayan}$$

$$\text{ceil}\left(\frac{\lambda}{\mu}\right) = 1$$

$$ORIGIN \equiv \frac{c_{min}(\lambda, \mu)}{\text{pelayan}}$$

$$\text{konstanta} \equiv 7$$

$$c_{atas}(\lambda, \mu) \equiv \text{konstanta } c_{min}(\lambda, \mu)$$

Jumlah pelayan paralel:

$$c := c_{min}(\lambda, \mu), (c_{min}(\lambda, \mu) + 1 \text{ pelayan}) .. c_{atas}(\lambda, \mu)$$

$$c_{min}(\lambda, \mu) = 1 \text{ pelayan}$$

$$c_{atas}(\lambda, \mu) = 7 \text{ pelayan}$$

Faktor utilisasi / intensitas lalu lintas:

$$\rho(\lambda, \mu, c) := \frac{\lambda}{c \mu}$$

Ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan c :

$$EOT_{GGcGD}(\lambda, \mu, c, O_1, O_2) = EOO_{GGcGD}(c, O_1) + EON_{GGcGD}(\lambda, \mu, c, O_2)$$

Ekspektasi ongkos operasi para pelayan per satuan waktu untuk jumlah pelayan c :

$$EOO_{GGcGD}(c, O_1) := \begin{cases} c O_1 & \text{if } 0 < \frac{\lambda}{\frac{c}{\text{pelayan}} \mu} < 1 \\ \text{"Tidak didefinisikan"} & \text{otherwise} \end{cases}$$

$c =$	$EOO_{GGcGD}(c, O_1)$
1 pelayan	$\frac{Rp}{jam}$
2	$3 \cdot 10^4$
3	$6 \cdot 10^4$
4	$9 \cdot 10^4$
5	$1.2 \cdot 10^5$
6	$1.5 \cdot 10^5$
7	$1.8 \cdot 10^5$
	$2.1 \cdot 10^5$

Batas bawah ekspektasi jumlah pelanggan dalam sistem:

$$EN_{GGcGD_BB}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if } \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if } \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} - \frac{(c - 1) ES^2}{2 c ES} \geq 0 \\ \left| \begin{array}{l} h1 \leftarrow \frac{ES^2 - 2 c ET ES}{2 c (c ET - ES)} \\ h2 \leftarrow \frac{(c - 1) ES^2}{2 c ES} \\ \lambda_{eff} [(h1 - h2) + ES] \text{ pelanggan} \\ \lambda_{eff} (0 + ES) \text{ pelanggan otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{array} \right. \end{cases}$$

Batas atas ekspektasi jumlah pelanggan dalam sistem:

$$EN_{GGcGD_BA}(\lambda, \mu, c) := \begin{cases} c \leftarrow \frac{c}{\text{pelayan}} \\ \lambda_{eff} \leftarrow \lambda \\ \text{if} \left(0 < \frac{\lambda}{c \mu} < 1 \right) \\ \left| \begin{array}{l} \text{if} \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \geq 0 \\ \left| \begin{array}{l} h \leftarrow \frac{c^2 VT + VS + (c - 1) ES^2}{2 c (c ET - ES)} \\ \lambda_{eff} (h + ES) \text{ pelanggan} \end{array} \right. \\ \lambda_{eff} \left(\infty \frac{\text{jam}}{\text{pelanggan}} \right) \text{ pelanggan otherwise} \end{array} \right. \\ \text{"Tidak didefinisikan" otherwise} \end{cases}$$

Median ekspektasi pelanggan sistem yaitu median rata-rata pelanggan dalam sistem besarnya (batas bawahnya + batas atasnya)/2:

$$EN_{GGcGD_Median}(\lambda, \mu, c) := \frac{EN_{GGcGD_BB}(\lambda, \mu, c) + EN_{GGcGD_BA}(\lambda, \mu, c)}{2}$$

Ekspektasi ongkos para pelanggan berada dalam sistem per satuan waktu untuk jumlah pelayan c :

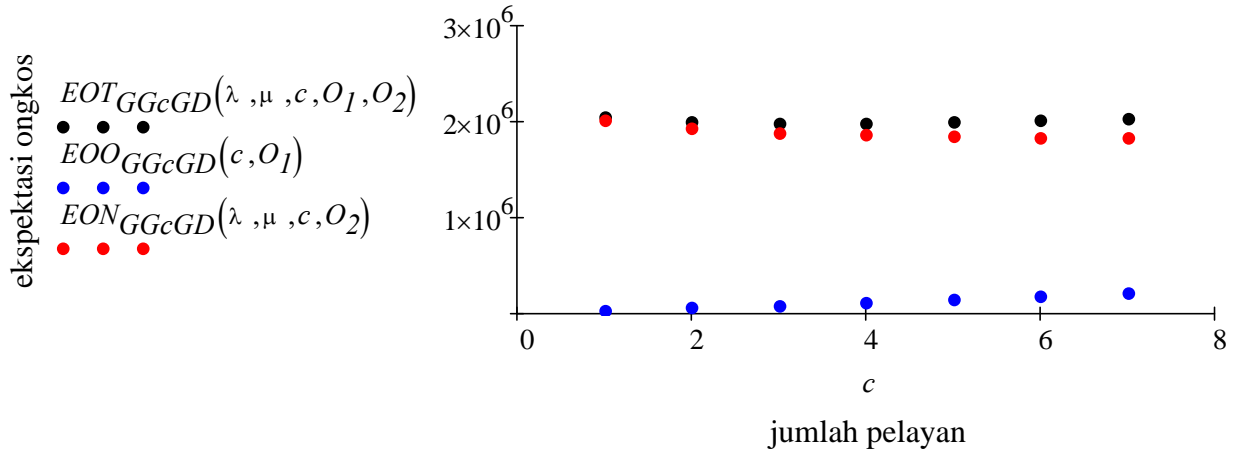
$$EON_{GGcGD}(\lambda, \mu, c, O_2) := O_2 EN_{GGcGD_Median}(\lambda, \mu, c)$$

$c =$		$EON_{GGcGD}(\lambda, \mu, c, O_2)$	$\frac{Rp}{jam}$
1	pelayan	$2.02083 \cdot 10^6$	
2		$1.93269 \cdot 10^6$	
3		$1.8851 \cdot 10^6$	
4		$1.85954 \cdot 10^6$	
5		$1.84375 \cdot 10^6$	
6		$1.83305 \cdot 10^6$	
7		$1.82533 \cdot 10^6$	

Ekspektasi ongkos total sistem antrian per satuan waktu untuk jumlah pelayan c :

$$EOT_{GGcGD}(\lambda, \mu, c, O_1, O_2) := EOO_{GGcGD}(c, O_1) + EON_{GGcGD}(\lambda, \mu, c, O_2)$$

$c =$		$EOO_{GGcGD}(c, O_1)$	$EON_{GGcGD}(\lambda, \mu, c, O_2)$	$EOT_{GGcGD}(\lambda, \mu, c, O_1, O_2)$
		$\frac{Rp}{jam}$	$\frac{Rp}{jam}$	$\frac{Rp}{jam}$
1	pelayan	$3 \cdot 10^4$	$2.02083 \cdot 10^6$	$2.05083 \cdot 10^6$
2		$6 \cdot 10^4$	$1.93269 \cdot 10^6$	$1.99269 \cdot 10^6$
3		$9 \cdot 10^4$	$1.8851 \cdot 10^6$	$1.97510 \cdot 10^6$
4		$1.2 \cdot 10^5$	$1.85954 \cdot 10^6$	$1.97954 \cdot 10^6$
5		$1.5 \cdot 10^5$	$1.84375 \cdot 10^6$	$1.99375 \cdot 10^6$
6		$1.8 \cdot 10^5$	$1.83305 \cdot 10^6$	$2.01305 \cdot 10^6$
7		$2.1 \cdot 10^5$	$1.82533 \cdot 10^6$	$2.03533 \cdot 10^6$



Kurva ekspektasi ongkos vs. jumlah pelayan

Jumlah pelayan optimal:

Dalam program ini $c_{atas} \leftarrow \text{konstanta } c_{min}$.

$\text{konstanta} = 7$

$$c_{optGGc}(\lambda, \mu, O_1, O_2) := \left\{ \begin{array}{l} c_{min} \leftarrow \left\{ \begin{array}{l} \text{ceil}\left(\frac{\lambda}{\mu}\right) + 1 \text{ if } \text{ceil}\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ \text{ceil}\left(\frac{\lambda}{\mu}\right) \text{ otherwise} \end{array} \right. \\ c_{atas} \leftarrow \text{konstanta } c_{min} \\ \text{for } c \in c_{min} \dots c_{atas} \\ v_{EOT}_c \leftarrow \left\{ \begin{array}{l} EOT_{GGcGD}(\lambda, \mu, c \text{ pelayan}, O_1, O_2) \text{ if } 0 < \frac{\lambda}{c \mu} < 1 \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{array} \right. \\ \left(\text{match}\left(\min(v_{EOT}), v_{EOT}\right)_{c_{min}} \right) \text{ pelayan} \end{array} \right.$$

$$c_{optGGc}(\lambda, \mu, O_1, O_2) = 3 \text{ pelayan}$$

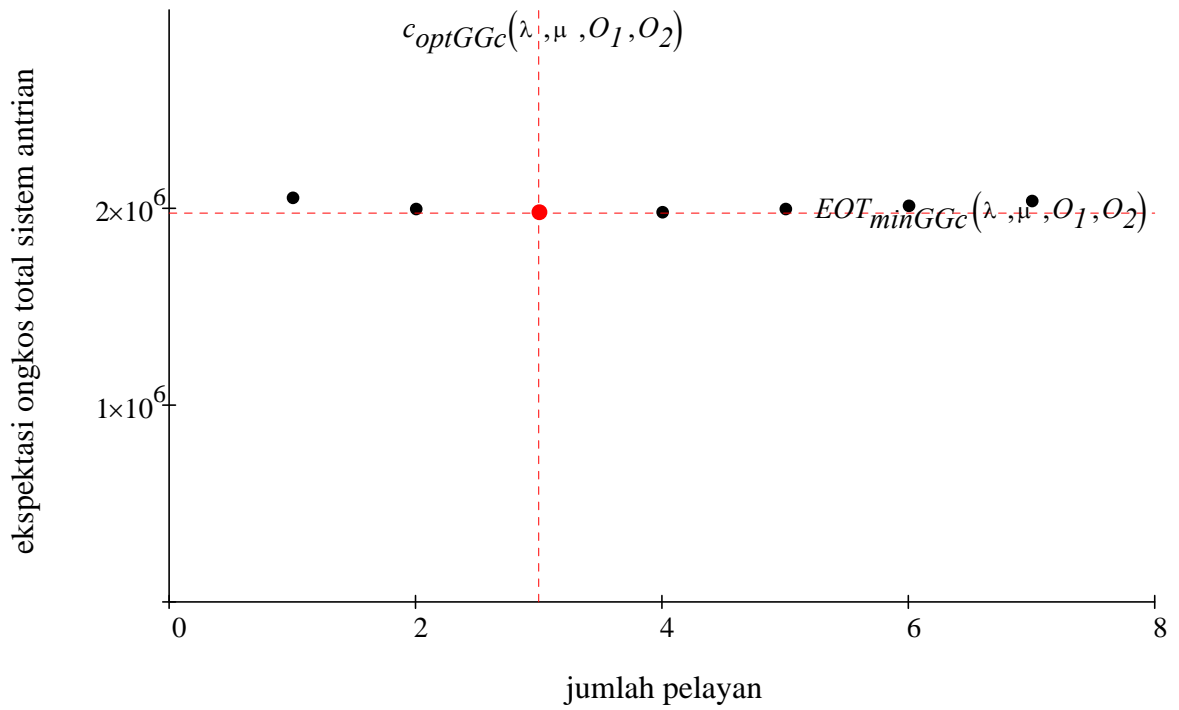
Ekspektasi ongkos total sistem antrian minimum:

Dalam program ini $c_{atas} \leftarrow \text{konstanta } c_{min}$.

$\text{konstanta} = 7$

$$EOT_{minGGc}(\lambda, \mu, O_1, O_2) := \left\{ \begin{array}{l} c_{min} \leftarrow \left\{ \begin{array}{l} \text{ceil}\left(\frac{\lambda}{\mu}\right) + 1 \text{ if } \text{ceil}\left(\frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} \\ \text{ceil}\left(\frac{\lambda}{\mu}\right) \text{ otherwise} \end{array} \right. \\ c_{atas} \leftarrow \text{konstanta } c_{min} \\ \text{for } c \in c_{min} \dots c_{atas} \\ v_{EOT}_c \leftarrow \left\{ \begin{array}{l} EOT_{GGcGD}(\lambda, \mu, c \text{ pelayan}, O_1, O_2) \text{ if } 0 < \frac{\lambda}{c \mu} < 1 \\ \text{"Tidak didefinisikan"} \text{ otherwise} \end{array} \right. \\ EOT_{min} \leftarrow \min(v_{EOT}) \end{array} \right.$$

$$EOT_{minGGc}(\lambda, \mu, O_1, O_2) = 1.9751 \times 10^6 \frac{Rp}{jam}$$



Kurva ekspektasi ongkos total sistem antrian vs. jumlah pelayan