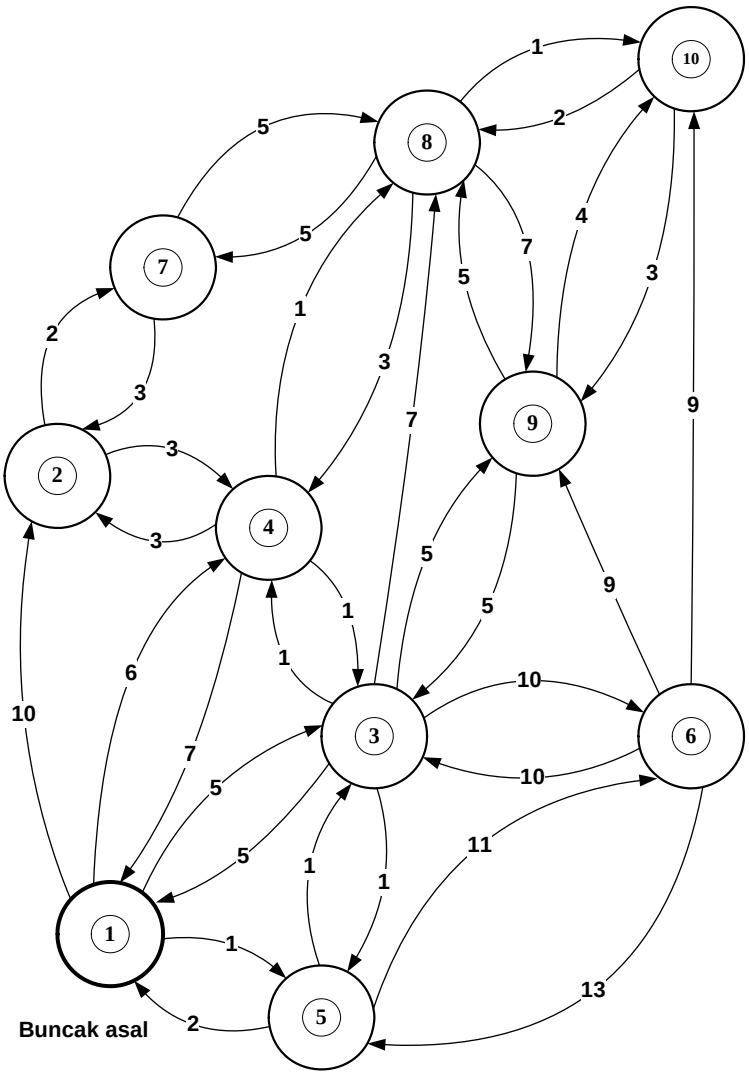
# *TSP - Travelling Salesperson Problem*

**Oleh:**

**Dr. Ir. H. Muhammad Sutarno, S.H.I., M.Sc., M.Ag.**

Perhatikan jaringkerja *TSP* di bawah ini. Mulai dari buncak 1 kembali ke buncak 1.



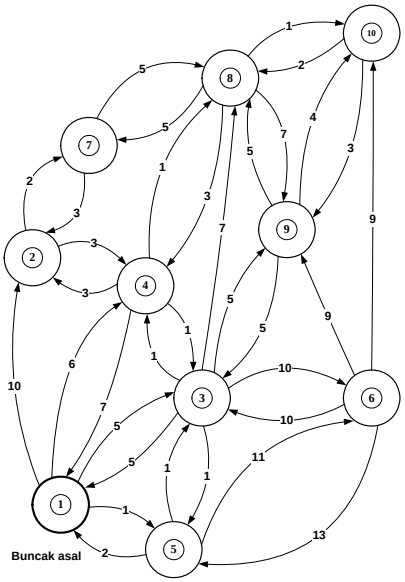
Buncak asal. $s := 1$ dan nomer buncak terbesar $t := 10$

**Persoalannya adalah mana lintas yang harus dilintasi dari buncak asal kembali ke buncak asal dan harus melewati semua buncak lainnya hanya sekali saja agar jarak tempuh totalnya minimum.**

Jaringkerja *TSP* di atas dapat dinyatakan dalam matriks kedekatan *X* (*adjacency matrix*) di bawah ini:

Matriks kedekatan (*adjacency matrix*):

$$X := \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$




Dari buncak 1 ada hubungan ke arah buncak 2, 3, 4, 5; dan hubungan dengan arah tersebut di beri nilai satu. Untuk yang tidak ada hubungan dari buncak 1 diberi nilai nol.

Contoh nilai kedekatan $X_{1,j}$ dari buncak 1 ke arah buncak *j*:

$X_{1,1} = 0$ $X_{1,6} = 0$

$X_{1,2} = 1$ $X_{1,7} = 0$

$X_{1,3} = 1$ $X_{1,8} = 0$

$X_{1,4} = 1$ $X_{1,9} = 0$ $X_{1,4} = 1$

$X_{1,5} = 1$ $X_{1,10} = 0$

Contoh nilai kedekatan $X_{3,j}$ dari buncak 3 ke arah buncak *j*:

$X_{3,1} = 1$ $\quad$ $X_{3,6} = 1$

$X_{3,2} = 0$ $\quad$ $X_{3,7} = 0$

$X_{3,3} = 0$ $\quad$ $X_{3,8} = 1$

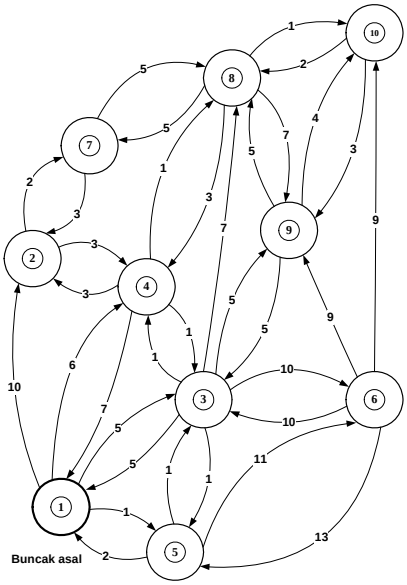
$X_{3,4} = 1$ $\quad$ $X_{3,9} = 1$

$X_{3,5} = 1$ $\quad$ $X_{3,10} = 0$

Matriks jarak (*distance matrix*) untuk jaring kerja *TSP* ini dari buncak *i* ke buncak *j dalam satuan jarak* seperti tampak di bawah ini:

$D_{i,j} = \infty$ $\quad$ $\infty := 10^9$ ("angka ini peka") $\quad$ untuk cabang "i,j" yang tidak ada

$$D := \begin{pmatrix} \infty & 10 & 5 & 6 & 1 & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 3 & \infty & \infty & 2 & \infty & \infty & \infty \\ 5 & \infty & \infty & 1 & 1 & 10 & \infty & 7 & 5 & \infty \\ 7 & 3 & 1 & \infty & \infty & \infty & \infty & 1 & \infty & \infty \\ 2 & \infty & 1 & \infty & \infty & 11 & \infty & \infty & \infty & \infty \\ \infty & \infty & 10 & \infty & 13 & \infty & \infty & \infty & 9 & 9 \\ \infty & 3 & \infty & \infty & \infty & \infty & \infty & 5 & \infty & \infty \\ \infty & \infty & \infty & 3 & \infty & \infty & 5 & \infty & 7 & 1 \\ \infty & \infty & 5 & \infty & \infty & \infty & \infty & 5 & \infty & 4 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2 & 3 & \infty \end{pmatrix}$$




Contoh:

$D_{1,4} = 6$

$D_{4,8} = 1$

$D_{8,4} = 3$

**Jawab persoalan di atas adalah mencari matriks alir *F* yang meminimumkan jarak tempuh total. Secara matematis dapat dirumuskan sebagai berikut:**

*minimize* $\sum_{i=1}^{rows(X)} \sum_{j=1}^{cols(X)} \left(D_{i,j} \ F_{i,j} \ X_{i,j}\right)$



dengan kendala/*Given*:

1. Semua aliran (*flow*) yang keluar dari buncak asal 1 yaitu $F_{1,j}$, $i = 1..t$ jumlahnya harus sama dengan satu.

$$\sum_{j=1}^{cols(X)} F_{1,j} = 1$$

2. Semua aliran (*flow*) yang masuk ke buncak asal 1 yaitu $F_{i,1}$, $i = 1..t$ jumlahnya harus sama dengan satu.

$$\sum_{i=1}^{rows(X)} F_{i,1} = 1$$

2. Konservasi alir:

$$\sum_{j=1}^{cols(X)} F_{k,j} = \sum_{i=1}^{rows(X)} F_{i,k}$$

$k \in HimpunBuncakAntara$

Di buncak antara:

Jumlah aliran yang keluar dari buncak antara = jumlah aliran yang masuk ke buncak antara

$$\sum_{j=1}^{cols(X)} F_{k,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,k} = 1$$

4. Angka setiap aliran *F* adalah biner 1 atau nol.

$s = 1$ $t = 10$

$i := 1..rows(X)$ $rows(X) = 10$

$j := 1..cols(X)$ $cols(X) = 10$

Nilai tamu $F$:

$$F := \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Fungsi tujuan minimisasi jarak tempuh total:

$$z(F) := \sum_{i=1}^{rows(X)} \sum_{j=1}^{cols(X)} \left(D_{i,j}\ F_{i,j}\ X_{i,j}\right)$$

$s = 1$ $t = 10$ $cols(X) = 10$ $rows(X) = 10$

*Given*

$$\sum_{j=1}^{cols(X)} F_{1,j} = 1 \qquad F_{1,1} = 0 \quad F_{1,6} = 0 \quad F_{1,7} = 0 \quad F_{1,8} = 0 \quad F_{1,9} = 0 \quad F_{1,10} = 0$$

$$\sum_{i=1}^{rows(X)} F_{i,1} = 1 \qquad \sum_{j=1}^{cols(X)} F_{1,j} = \sum_{i=1}^{rows(X)} F_{i,1}$$

$$\sum_{j=1}^{cols(X)} F_{2,j} = \sum_{i=1}^{rows(X)} F_{i,2} \qquad F_{2,1} = 0 \quad F_{2,2} = 0 \quad F_{2,3} = 0 \quad F_{2,5} = 0 \quad F_{2,6} = 0 \quad F_{2,8} = 0 \quad F_{2,9} = 0 \quad F_{2,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{2,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,2} = 1$$

$$\sum_{j=1}^{cols(X)} F_{3,j} = \sum_{i=1}^{rows(X)} F_{i,3} \qquad F_{3,2} = 0 \quad F_{3,3} = 0 \quad F_{3,7} = 0 \quad F_{3,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{3,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,3} = 1$$

$$\sum_{j=1}^{cols(X)} F_{4,j} = \sum_{i=1}^{rows(X)} F_{i,4} \qquad F_{4,4} = 0 \quad F_{4,5} = 0 \quad F_{4,6} = 0 \quad F_{4,7} = 0 \quad F_{4,9} = 0 \quad F_{4,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{4,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,4} = 1$$

$$\sum_{j=1}^{cols(X)} F_{5,j} = \sum_{i=1}^{rows(X)} F_{i,5} \qquad F_{5,2} = 0 \quad F_{5,4} = 0 \quad F_{5,5} = 0 \quad F_{5,7} = 0 \quad F_{5,8} = 0 \quad F_{5,9} = 0 \quad F_{5,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{5,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,5} = 1$$

$$\sum_{j=1}^{cols(X)} F_{6,j} = \sum_{i=1}^{rows(X)} F_{i,6} \qquad F_{6,1} = 0 \quad F_{6,2} = 0 \quad F_{6,4} = 0 \quad F_{6,6} = 0 \quad F_{6,7} = 0 \quad F_{6,8} = 0$$

$$\sum_{j=1}^{cols(X)} F_{6,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,6} = 1$$

$$\sum_{j=1}^{cols(X)} F_{7,j} = \sum_{i=1}^{rows(X)} F_{i,7} \qquad F_{7,1} = 0 \quad F_{7,3} = 0 \quad F_{7,4} = 0 \quad F_{7,5} = 0 \quad F_{7,6} = 0 \quad F_{7,7} = 0 \quad F_{7,9} = 0 \quad F_{7,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{7,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,7} = 1$$

$$\sum_{j=1}^{cols(X)} F_{8,j} = \sum_{i=1}^{rows(X)} F_{i,8} \qquad F_{8,1} = 0 \quad F_{8,2} = 0 \quad F_{8,3} = 0 \quad F_{8,5} = 0 \quad F_{8,6} = 0 \quad F_{8,8} = 0$$

$$\sum_{j=1}^{cols(X)} F_{8,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,8} = 1$$

$$\sum_{j=1}^{cols(X)} F_{9,j} = \sum_{i=1}^{rows(X)} F_{i,9} \qquad F_{9,1} = 0 \quad F_{9,2} = 0 \quad F_{9,4} = 0 \quad F_{9,5} = 0 \quad F_{9,6} = 0 \quad F_{9,7} = 0 \quad F_{9,9} = 0$$

$$\sum_{j=1}^{cols(X)} F_{9,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,9} = 1$$

$$\sum_{j=1}^{cols(X)} F_{10,j} = \sum_{i=1}^{rows(X)} F_{i,10} \qquad F_{10,1} = 0 \quad F_{10,2} = 0 \quad F_{10,3} = 0 \quad F_{10,4} = 0 \quad F_{10,5} = 0$$

$$F_{10,6} = 0 \quad F_{10,7} = 0 \quad F_{10,10} = 0$$

$$\sum_{j=1}^{cols(X)} F_{10,j} = 1$$

$$\sum_{i=1}^{rows(X)} F_{i,10} = 1$$

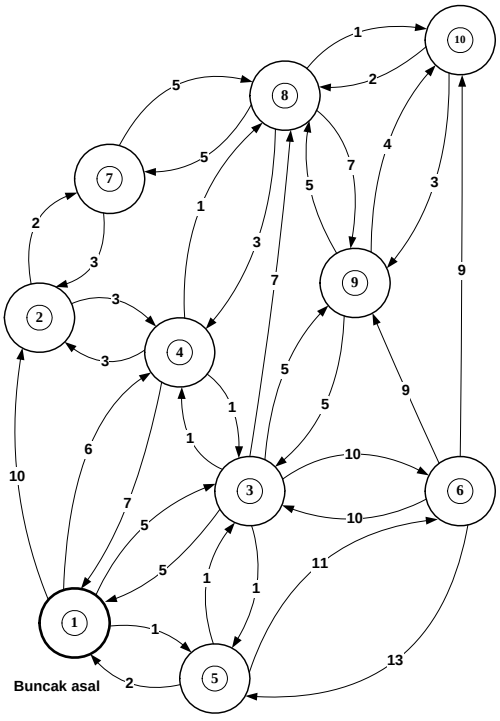
$$F_{5,1} = 0 \qquad F_{2,7} = 0$$

$$F \geq 0$$

$$F := Minimize(z, F)$$

$F =$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |




$$z(F) = 44$$

Lintas *TSP*-nya dalam bentuk vektor:

$$
\begin{aligned}
&LTSP(F) := \\
&\quad k \leftarrow 1 \\
&\quad m \leftarrow s \\
&\quad VL_1 \leftarrow s \\
&\quad \textit{while}\ \ 1 \\
&\qquad k \leftarrow k + 1 \\
&\qquad VL_k \leftarrow match\left[1, \left(F^T\right)^{\langle m \rangle}\right]_1 \\
&\qquad m \leftarrow match\left[1, \left(F^T\right)^{\langle m \rangle}\right]_1 \\
&\qquad \textit{break}\ \ \textit{if}\ \ m = 1 \\
&\quad VL
\end{aligned}
$$

Lintas *TSP*-nya dalam bentuk vektor :

$LTSP(F) =$

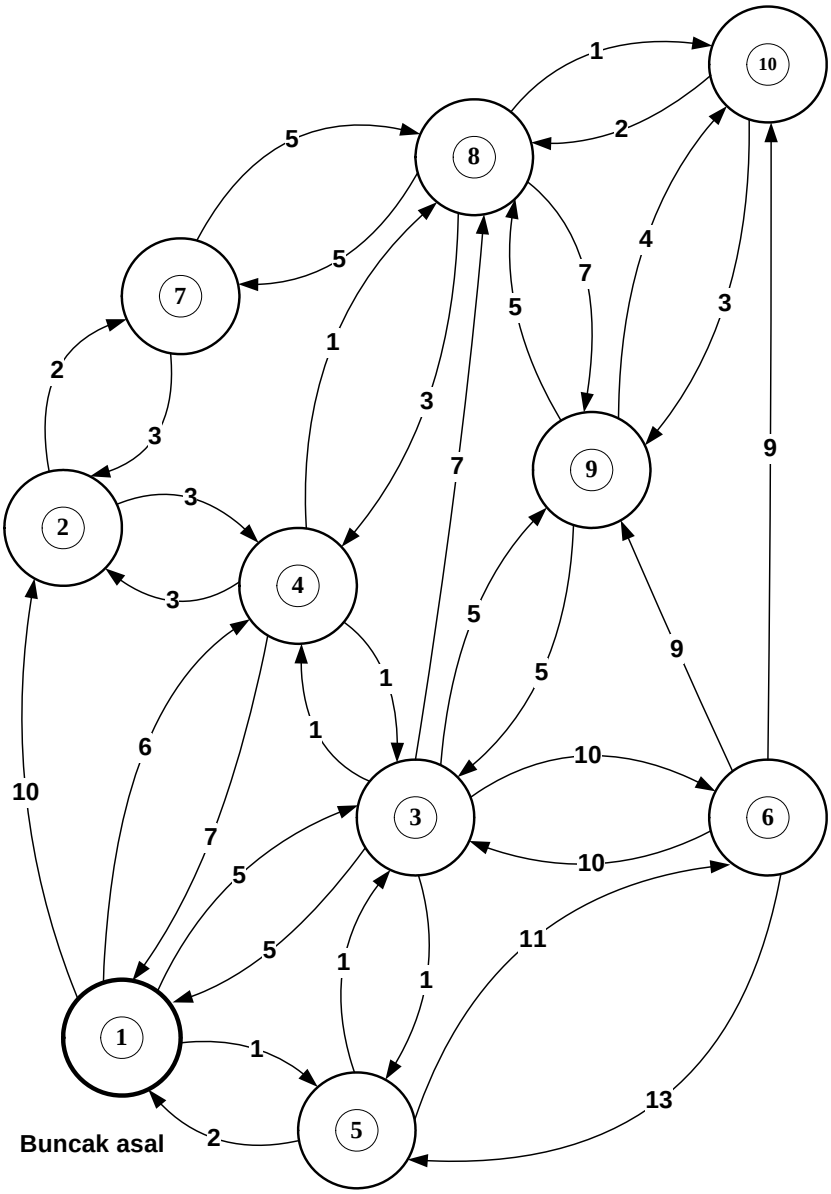
| | 1 |
|---|---|
| 1 | 1 |
| 2 | 5 |
| 3 | 6 |
| 4 | 9 |
| 5 | 10 |
| 6 | 8 |
| 7 | 7 |
| 8 | 2 |
| 9 | 4 |
| 10 | 3 |
| 11 | 1 |

$$\begin{pmatrix} 1 \\ 5 \\ 6 \\ 9 \\ 10 \\ 8 \\ 7 \\ 2 \\ 4 \\ 3 \\ 1 \end{pmatrix}$$

Nilai lintas paling pendek :

$NTSP(F) := z(F)$

$NTSP(F) = 44$




$LTSP(F) =$

| | 1 |
|---|---|
| 1 | 1 |
| 2 | 5 |
| 3 | 6 |
| 4 | 9 |
| 5 | 10 |
| 6 | 8 |
| 7 | 7 |
| 8 | 2 |
| 9 | 4 |
| 10 | 3 |
| 11 | 1 |