

PROBABILITAS DAN STATISTIKA

FUNGSI DISTRIBUSI VARIABEL ACAK

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Definisi 4.1. Fungsi Distribusi Variabel Acak Diskrit

Fungsi distribusi $F_X(x)$ dari variabel acak diskrit X adalah probabilitas untuk nilai variabel acak diskrit X lebih kecil atau sama dengan x . Ditulis

$$F_X(x) = P(X \leq x) = P[\omega \mid (X(\omega) \leq x)] = \sum_{(x_i \leq x)} p(x_i)$$

Fungsi distribusi $F_X(x)$ ini, biasa juga ditulis dengan $F(x)$ atau lebih singkat F .

Contoh 4.1

Ruang sampel

$$\Omega := \begin{pmatrix} \text{"RR"} \\ \text{"RB"} \\ \text{"BR"} \\ \text{"BB"} \end{pmatrix} \quad \omega := \Omega \quad r := 1..rows(\Omega) \quad rows(\Omega) = 4$$

$$\Omega = \begin{pmatrix} \text{"RR"} \\ \text{"RB"} \\ \text{"BR"} \\ \text{"BB"} \end{pmatrix} \quad \omega_r = \begin{pmatrix} \text{"RR"} \\ \text{"RB"} \\ \text{"BR"} \\ \text{"BB"} \end{pmatrix}$$

$$X(\text{hasil}) := \begin{cases} 0 & \text{if hasil = "RR"} \\ 1 & \text{if hasil = "RB"} \\ 1 & \text{if hasil = "BR"} \\ 2 & \text{if hasil = "BB"} \end{cases}$$

$$X("RR") = 0$$

$$X("RB") = 1$$

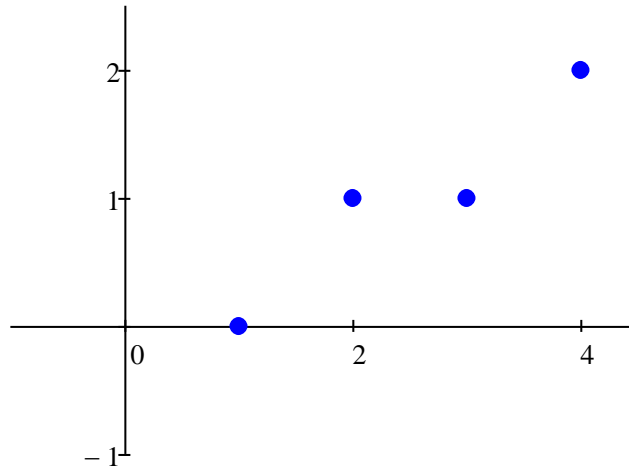
$$X("BR") = 1$$

$$X("BB") = 2$$

$$X(\omega_r) =$$

0
1
1
2

$X(\omega_r)$

$$X_r := X(\omega_r)$$

$$X := X$$

$$X = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

Variabel acak X

$$X(\omega_r) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\text{rows}(X) = 4$$

misal

$$p1 := 1 \times 10^{-4} \quad p2 := 9.9 \times 10^{-3} \quad p3 := 9.9 \times 10^{-3} \quad p4 := 0.9801$$

$$Prob(\text{nilai}X_{\omega}) := \begin{cases} p1 & \text{if } \text{nilai}X_{\omega} = X(\omega_1) \\ p2 & \text{if } \text{nilai}X_{\omega} = X(\omega_2) \\ p3 & \text{if } \text{nilai}X_{\omega} = X(\omega_3) \\ p4 & \text{if } \text{nilai}X_{\omega} = X(\omega_4) \end{cases}$$

$\omega_r = \begin{pmatrix} \text{"RR"} \\ \text{"RB"} \\ \text{"BR"} \\ \text{"BB"} \end{pmatrix}$	$X(\omega_r) =$	$Prob(X(\omega_r)) =$
	0	$1 \cdot 10^{-4}$
	1	$9.9 \cdot 10^{-3}$
	1	$9.9 \cdot 10^{-3}$
	2	0.9801

$R(X) := Set(X)$ Ruang rentang X

$$R(X) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$i := 1..rows(R(X))$

$$R(X)_i = \begin{matrix} \boxed{0} \\ \boxed{1} \\ \boxed{2} \end{matrix} \quad \omega = \begin{pmatrix} \text{"RR"} \\ \text{"RB"} \\ \text{"BR"} \\ \text{"BB"} \end{pmatrix}$$

$x_i := R(X)_i$

$$x = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

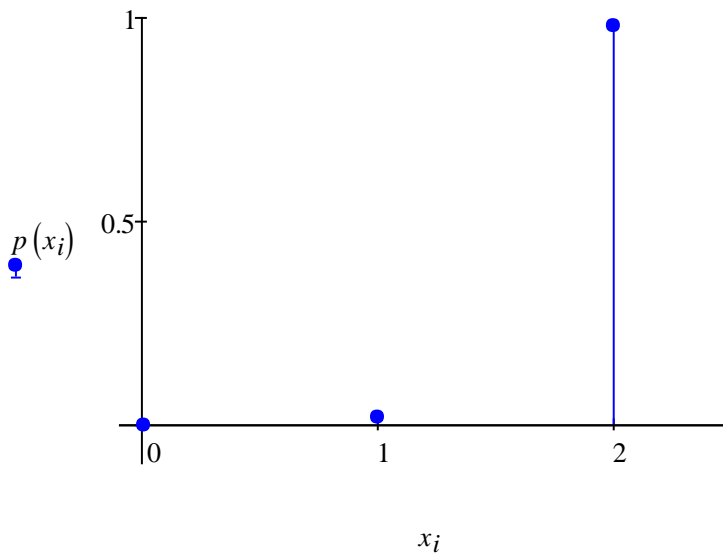
$$X(\omega_r) = \begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline 1 \\ \hline 2 \\ \hline \end{array} \quad X = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad R(X) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad R(X)_i = \begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline 2 \\ \hline \end{array} \quad x_i = \begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline 2 \\ \hline \end{array}$$

```

p(nilai_x) :=
  t ← 1
  for i ∈ 1..last(X)
    if X_i = nilai_x
      V_t ← X_i
      T ← t
      t ← t + 1
  sum_{t=1}^T Prob(V_t)
  
```

Nilai-nilai X dan probabilitasnya

$x_i =$	$p(x_i) =$	$p(R(X)_i) =$
0	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
1	0.0198	0.0198
2	0.9801	0.9801



Probabilitas kumulatif

```

F(nilai_x) :=
  t ← 1
  for i ∈ 1..last(X)
    if X_i ≤ nilai_x
      V_t ← X_i
      T ← t
      t ← t + 1
  ∑_{t=1}^T Prob(V_t)
  
```

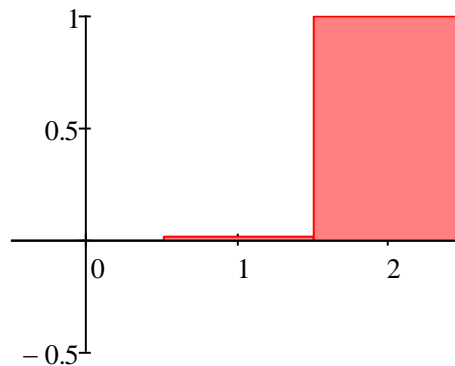
$x_i =$	$p(x_i) =$	$R(X)_i =$	$F(R(X)_i) =$
0	1·10 ⁻⁴	0	1·10 ⁻⁴
1	0.0198	1	0.0199
2	0.9801	2	1

$R_X := R(X)$ $R_X = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$F(R(X)_i) =$

1·10 ⁻⁴
0.0199
1

$F(R(X)_i)$

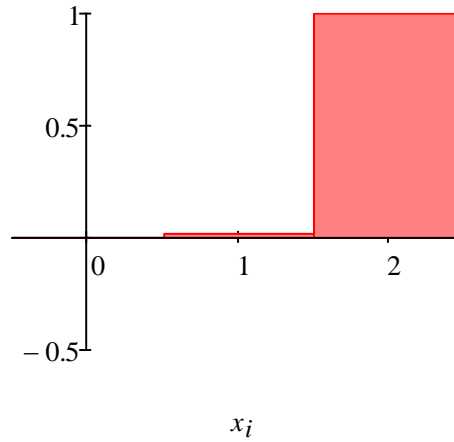


x_i

Nilai-nilai X dan fungsi distribusinya

$x_i =$	$F(x_i) =$
0	$1 \cdot 10^{-4}$
1	0.0199
2	1

$F(x_i)$



Contoh 4.2

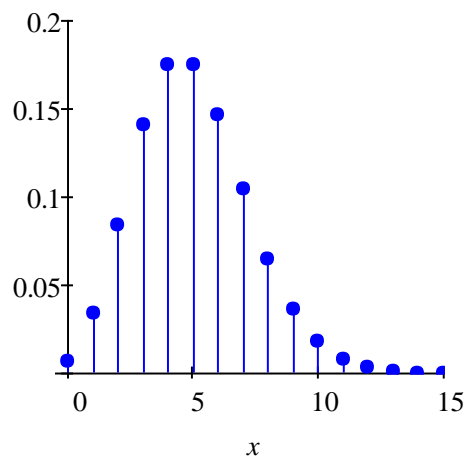
$x := 0.. \infty$

$$p(x) := \frac{5^x e^{-5}}{x!}$$

$$\sum_{x=0}^{\infty} p(x) = 1$$

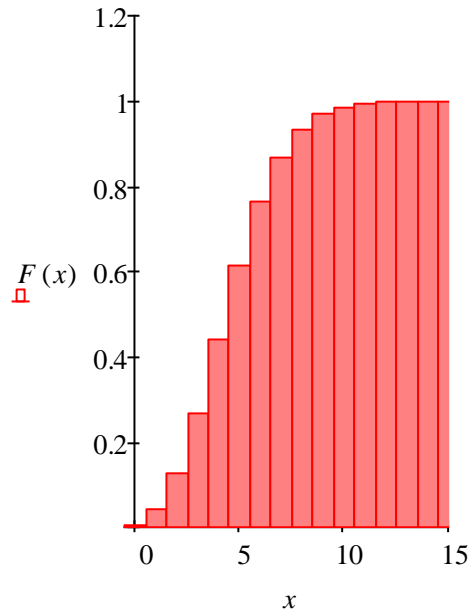
$x =$	$p(x) =$
0	$6.738 \cdot 10^{-3}$
1	0.034
2	0.084
3	0.140
4	0.175
5	0.175
6	0.146
7	0.104
8	0.065
9	0.036
10	0.018
11	$8.242 \cdot 10^{-3}$
12	$3.434 \cdot 10^{-3}$
...	...

$p(x)$



$$F(x) := \sum_{i=0}^x p(i)$$

$x =$	$F(x) =$
0	$6.738 \cdot 10^{-3}$
1	0.040
2	0.125
3	0.265
4	0.440
5	0.616
6	0.762
7	0.867
8	0.932
9	0.968
10	0.986
11	0.995
12	0.998
...	...



Definisi 4.2. Fungsi Distribusi Variabel Acak Kontinu

Fungsi distribusi $F_x(x)$ dari variabel acak kontinu X adalah probabilitas untuk nilai variabel acak kontinu X lebih kecil atau sama dengan x . Ditulis

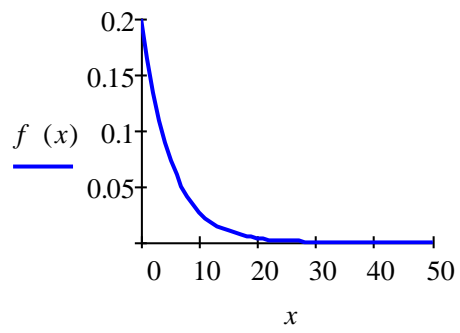
$$F_x(x) = P(X \leq x) = P[\omega \mid (X(\omega) \leq x)] = \int_{-\infty}^x f(u) du$$

di mana $f(x)$ adalah fungsi padat probabilitas X .

Fungsi distribusi $F_x(x)$ ini, biasa juga ditulis dengan $F(x)$ atau lebih singkat F .

Contoh 4.3

$$x := 0.. \infty \quad f(x) := \frac{1}{5} e^{-\frac{1}{5}x}$$



$$F(x) := \int_0^x \frac{1}{5} e^{-\frac{1}{5}u} du$$

